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Influence of Hall current and microrotation on the boundary layer flow of an electrically conducting fluid: Application to Hemodynamics

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ABSTRACT

In this study, a problem of micropolar fluid flow is analyzed taking into account the effect of Hall current. An external magnetic field is applied transverse to that of fluid flow. The partial differential equations of conservation of mass, linear momentum, angular momentum, energy and concentration are considered along with suitable boundary conditions in formulating the model for the physical problem and analyzing it theoretically. The problem is motivated towards exploring some novel information in the dynamics of blood flow, when the influence of Hall current and rotation of the microparticles of blood, for e.g. the erythrocytes and thrombocytes co-exist at the same platform. The governing equations are reduced to a system of non-linear ordinary differential equations by making use of similarity transformations. The resulting system of coupled non-linear ordinary differential equations is then solved by using a suitable numerical technique that involves the use of finite differences and Newton's linearization method. A parametric study illustrating the influences of the magnetic field, Hall current and the micropolarity of suspended microparticles has been carried out to investigate the variations in velocity, temperature and concentration profiles. The problem has an important bearing on some bio-engineering problems, where the biological conduits, cells and membranes are typically surrounded by fluids, which are electrically conducting (as in the case of blood) and the conduits, cells and membranes that are stretched constantly. The computational results reveal that the microrotation of erythrocytes in blood is enhanced due to the effect of Hall current, when blood flows in the arterial system.

1. Introduction

Boundary layer flows of incompressible viscous fluids in continuously moving surfaces have been studied by some researchers. Such studies have important applications in engineering and industry. Crane [1] remarked that results of these studies can be applied to situations, where the fluid flows over stretching sheets. In the realm of physiological fluid dynamics, it is known that the physical structure and mechanical properties of blood vessels largely depend on variation of pressure and other flow conditions. Hence for a complete analysis of complex flow problems of blood flow in arteries, such as cerebral and carotid arteries, it is essential to have detailed information of the transport of blood-like fluids over stretching sheets. Thus studies of flow on stretching sheets bear the prospect of applications in biomedical engineering too.

A problem of magnetohydrodynamic stagnation point flow and heat transfer was recently solved numerically by Agbaje et al. [2]. Unsteady flow of blood in arteries was investigated by Misra and Sahu [3], by considering cross-sectional non-uniformity of the endothelium. Mondal et al. [4] examined the effects of thermal radiation on MHD stagnation-point flow over a shrinking sheet for a situation, when the thermal conductivity varies with temperature. MHD boundary layer flow and heat transfer of an electrically conducting power-law fluid over a stretching surface in presence of a stagnation point, was analyzed by Ray Mahapatra et al. [5], by taking into account thermal radiation and suction/injection. The governing equations were solved numerically by using shooting method. They observed that the temperature of the fluid at a given point increases as the magnetic field intensity increases, when the stretching velocity exceeds the free stream velocity. A problem of unsteady double diffusive convection was solved by Mondal and Sibanda [6], by considering non-uniform boundary conditions. Unsteady nanofluid flow over a stretching sheet for a Casson fluid was analyzed by Oyelakin et al. [7], by accounting for thermal radiation. Several other studies on nanofluid flow on stretching/shrinking surfaces for different

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situations were performed by Haroun et al. [8–10], by using spectral relaxation method, while Ahamed et al. [11] studied magneto-nanofluid flow, by considering thermo-diffusion effects and viscous dissipation.

It is known that Navier-Stokes equations are inadequate to account for the microscopic manifestations of the complex hydrodynamical behavior of many industrial fluids and also some physiological fluids, like blood. All these fluids exhibit non-Newtonian behavior. Keeping this in mind, different researchers have developed various non-Newtonian models. The choice of one particular model should be made depending on the aim of a particular study. The theory of micropolar fluids developed by Eringen [12–14] was used by some researchers in cases where effects of microrotation of the fluid particles in microscopic form are predominant. Blood contains microparticles, like erythrocytes (red blood cells), platelets. In order to account for the rotational motion of these microparticles, micropolar fluid theory can be used appropriately (cf. Turk et al. [15], Misra and Ghosh [16]).

The Hall effect reforms to the generation of an electric potential in a direction normal to an electric current that flows along a conducting material and a magnetic field applied externally. In recent developments of magnetohydrodynamic (MHD), strong magnetic fields are applied, so that the electromagnetic force can exert considerable influence. In the case of electrically conducting fluids flowing under the action of an externally applied magnetic field, the effects of Hall current is quite considerable, if the strength of the magnetic field is large. It may be mentioned that the problems of electrically conducting fluid flows in the presence of strong magnetic fields arise also in Geophysics, biology and medicine (particularly cardiac MRI, ECG, etc.).

In view of this, the effects of Hall current have been investigated for different fluid dynamical problems by some researchers. Watanabe and Pop [17] studied the effects of Hall current on magnetohydrodynamic boundary layer flow, while in the case of free convection flows, Hall effects were explored by Aboeldahab and Elbarbary [18]. In the case of Burgers' fluid, Hartmann flow and heat transfer were studied by Rana et al. [19] by considering Hall effects and for Oldroyd B-fluid, effect of Hall current on unsteady magnetohydrodynamic flow was examined by Asghar et al. [20].

Various aspects of arterial blood flow in different situations, including cases in which the artery is under the action of magnetic fields were studied by Misra et al. [21–41]. Some of these studies were carried out under normal physiological conditions, while some were conducted for blood flow in pathological fluids other than blood. In [33], the authors have discussed peristaltic flow of bile within ducts in the presence of bile stones. Different mathematical models were developed in different studies, depending on the objectives of the study. Misra and Sinha [42] investigated the effects of Hall current on fluid flow through a porous medium under the action of an external magnetic field, taking into account heat radiation, while Hayat et al. [43] examined the effects of Hall current on flow of a Maxwell fluid, using modified Darcy's law, when the flow takes place peristaltically.

To the best of the authors' knowledge, Hall current on fluid flow in the presence of rotation of the micro-molecules of the fluid has not been studied by any previous researchers. Keeping this in mind, the present study has been devoted to an investigation of the effects of Hall current on free convection flow of a magnetohydrodynamic flow and mass transfer, by accounting for the rotation of the microelements of the fluid. In the study of Hemodynamics, these micro-elements may be conceived of as the micro-particles like erythrocytes, platelets and thrombocytes that are suspended in blood. Thus although the study refers to fluids in general, it is primarily motivated towards finding an estimate of the extent of the influence of Hall current on the microrotation of the suspended particles of blood. The sheet over which the flow takes place is under a stretching motion. In hemodynamical studies, the micro-vessels like arterioles are known to assume the form of sheets, which are porous and also execute stretching motion. The present study refers to a situation, where the system is subject to the action of an external magnetic field of considerable strength in the transverse direction. The problem has been solved by restoring to a suitable numerical technique that involves the use of finite differences.

The numerical estimates of the different variables involved in the study have been computed and presented graphically. By using the graphs, variations in velocity, heat and mass transfer rates, rotation of the microelements have been discussed in detail, by considering different values of the parameters that depict the variation in Hall effect, magnetic field strength and micropolarity of the fluid. The results of the study bear the promise of applications to the flow of a variety of industrial and physiological fluids. It can also be applied to explore important information of many bioengineering problems, where the flexible surfaces of the biological conduits in living systems are in a state of stretching motion and are subject to the action of an external magnetic field. The present study will also be useful in validating the results of future investigations of more complex problems related to the behavior of ionic liquids and biomolecules.

2. The model

The model for the present study has been developed on the basis of the following assumptions:

(i) The fluid is incompressible, viscous and electrically conducting.
(ii) Rotational motion of the micro-elements of the fluid is predominant and it affects the fluid flow.
(iii) The flow takes place under the action of an external magnetic field B_0, whose intensity is considerably high and which acts along a direction normal to that flow.
(iv) The surface through which fluid flow takes place is flat and a slit-like structure is formed in the horizontal direction.
(v) The slit is stretched with a velocity, whose magnitude is directly proportional to the distance from a fixed point O.
(vi) The flow field is considerably affected by Hall current.
(vii) The species concentration and temperature of the fluid are maintained constant throughout the sheet.

We take O as the origin of coordinates and (x, y, z) as the rectangular frame of reference, the axis of x being taken along the direction of flow and y-axis along the direction normal to the stretching surface and z-axis is taken along the direction perpendicular to the xy-plane (cf. Fig. 1). The species concentration and the temperature are supposed to be maintained constant at the sheet and are denoted by C_w, T_w whose values at any large distance from the sheet can be put as

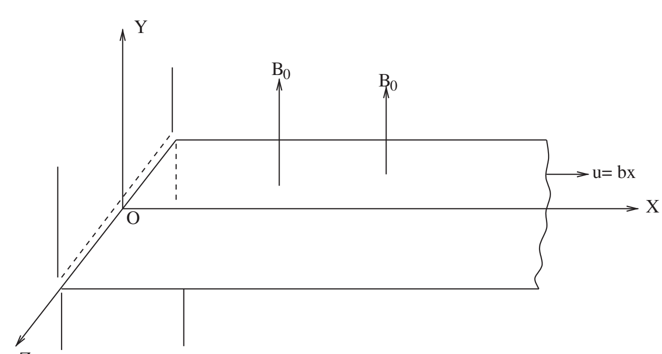

Fig. 1. Geometry of the problem.