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Electroosmotic flow through a microtube with sinusoidal roughness

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ABSTRACT

In this paper, a perturbation method is introduced to study the electroosmotic flow (EOF) in a microtube with slightly corrugated wall. The corrugation of the wall is described as periodic sinusoidal wave with small amplitude. Based on linearized Poisson–Boltzmann equation and the Cauchy momentum equation, the perturbation solutions for velocity and volume flow rate are obtained. The influences of the amplitude δ, the wave number λ, the nondimensional electrokinetic width K and the nondimensional pressure gradient G on the velocity and flow rate are analyzed graphically and discussed in detail. The results show that the flow rate Q of the EOF through a corrugated channel tends to the flow rate Q_0 of the EOF through a smooth channel when amplitude δ tends to zero, but the flow rate Q is always smaller than the flow rate Q_0 in the smooth channel. The flow retardation of the roughness on the flow rate of the EOF always increases with the augment of the nondimensional electrokinetic width K.

Nomenclature

a	mean radius of channel, m
e	elementary charge $= 1.602 \times 10^{-19}$ C
E_0	electric field strength, V m^{-1}
G	normalized axial pressure gradient $= -a^2/(\mu U_{eo})\partial p/\partial z$, —
K	dimensionless electrokinetic width $= \kappa a$, —
k_b	Boltzmann constant $= 1.38 \times 10^{-23}$ J K^{-1}
n_0	ion density of bulk liquid, m^{-3}
P	pressure, Pa
Q	dimensionless flow rate in rough channel, —
Q_0	dimensionless flow rate in smooth channel, —
R, θ, Z	cylindrical polar coordinate components, m, —, m
r, θ, z	dimensionless cylindrical polar coordinate components, —
T	absolute temperature, K
U_{eo}	Helmholtz–Smoluchowski electroosmotic velocity $= -\varepsilon\zeta E_0/\mu$, m s^{-1}
w	dimensionless axial velocity, —
z_ν	valence of ion, —

Greek symbols

δ	ratio of the corrugation amplitude to the mean radius of the channel, —
ε	dielectric constant, C V^{-1} m^{-1}
ζ	zeta potential, V
κ	inverse Debye length, m^{-1}
λ	wavenumber of wall corrugations, —
μ	dynamic viscosity, Pa·s
ρ_e	net volumetric charge density, C m^{-3}
ψ	electric potential, V
φ	dimensionless electric potential, —

1. Introduction

Microfluidic devices have become important due to its potential applications in physical and biochemical analysis [1,2]. Based on the actuation mechanism, various micropumps [3] such as electrohydrodynamic micropumps, electroosmotic micropumps, magnetohydrodynamic pumps, centrifugal pumps, reciprocating displacement micropumps have been developed. This paper presents the electroosmotic flow (EOF) in a microtube with slightly corrugated wall. Electroosmotic micropump is one of the important microfluidic systems which can transport fluids through microchannels without mechanical moving parts. The principle of electroosmotic flow is as follows. Generally, most solid surfaces will acquire a negative electric charge when brought into contact with a fluid containing dissociated salts. The charged surface will be able to influence the distribution of nearby ions in the solution and the outcome is the formation of an electric double layer (EDL). The EDL is a region close to the charged surface in which there is an excess of counterions over co-ions to neutralize the surface charge. When an axial electric field is applied, these ions will move. Due to viscous drag, the liquid is drawn by the ions and therefore flows through the channel. This kind of flow is known as EOF. The EOF of Newtonian fluids in various smooth microchannels has been studied theoretically, numerically and experimentally, by many researchers [4–19]. In addition, the flow and

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heat transfer of nanofluid and micropolar fluid have been investigated by Ganji et al. [20–25].

In practice, there are many roughnesses on surfaces of real channel walls, because the fabrication process or the adsorption of other species such as macromolecules can cause roughnesses on the wall of channel. A number of researchers studied the influence of wall roughness on laminar flow since 1970s. Wang [26] studied the effect of roughness on stokes flow between corrugated plates. The effects of surface wavy roughness on the fluid flow inside annuli with microfabricated solid walls were investigated by Chu [27]. Malevich et al. [28] theoretically studied three dimensional (3D) Couette flows between a plane and a wavy wall. Ng and Wang [29], by using the perturbation analysis method, analyzed the effect of corrugations on the Darcy–Brinkman flow through a porous channel with slightly corrugated walls. Qiao [30] investigated the effects of molecular level surface roughness on the EDL and the EOF by using molecular dynamics simulations. Hu et al. [31] studied theoretically and experimentally the EOF in slit microchannels with rectangular 3D prismatic elements fabricated on the bottom channel wall. Kang and Suh [32] investigated numerically two-dimensional electroosmotic flows in a microchannel with dielectric walls of rectangle-waved surface roughness. The effects of roughness element height and density on the EOF behavior are investigated by Yang and Liu [33]. Yau et al. [34] investigated the flow characteristics of EOF in a microchannel with complex wavy surfaces by the general method of coordinate transformation. Shu et al. [35] studied EOF through a microparallel channel with corrugated wavy walls under the Debye–Hückel approximation. They investigated the effects of corrugations on the EOF by performing perturbation analysis for small-amplitude corrugations as well as by applying the Ritz method and finite difference method for corrugations up to an amplitude of 0.5. Buren et al. [36] utilized perturbation method to investigate the electromagnetohydrodynamic (EMHD) flow in a microparallel channel with slightly corrugated walls. The 2D EMHD flow in a micro-parallel channel with slightly transverse corrugated walls is investigated by Buren and Jian [37]. In addition, Sheikholeslami et al. [38–40] numerically studied the effects of electric filed on the heat transfer and flow characteristic of a nanofluid in complex geometries.

The present article is an extension of reference [35]. In Ref. [35], the EOF through a corrugated parallel microchannel is studied when the zeta potentials on the upper and lower walls equal an identical constant and the cross-section does not change along the flow direction. However, there has been no published work about the EOF through a microtube with corrugated walls. The purpose of the present article is to investigate the EOF through a microtube with sinusoidally corrugated walls. The rest of the paper is presented as follows. In the second section of this article, the governing equations of EOF subjected to wavy boundary conditions are derived, and the approximate solutions for the electric potential, the velocity and the volume flow rate are obtained by using the perturbation method. In the third section, the influences of dimensional parameters on the EOF through a corrugated microtube are discussed in detail. Finally, conclusions are presented in the forth section.

2. Formulation of the problem

The steady EOF of an incompressible, viscous and electrolyte conducting Newtonian fluid in a microtube with sinusoidally wavy wall is considered. The geometry of problem and selection of coordinate system are shown in Fig. 1. The mean radius of the microtube is a. A cylindrical polar coordinate system (R, θ, Z) is introduced, where Z-axis is flow direction. The wavy wall is described by

$$R_w = a[1 + \delta \sin(\lambda\theta)], \tag{1}$$

where δ is the ratio of the corrugation amplitude to the mean radius of the channel and λ is the wavenumber of the corrugations. The small amplitude δ will be used as the perturbation parameter for the problems

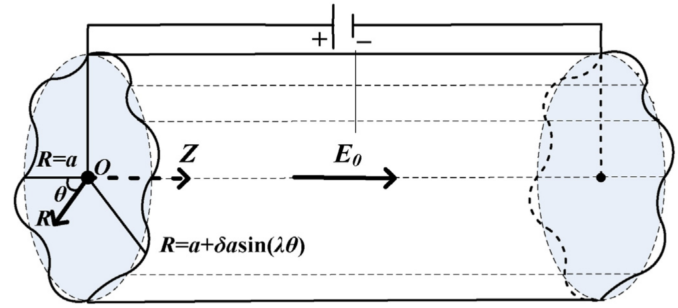

Fig. 1. Sketch of EOF through a circular micropipe with sinusoidal wavy microchannel.

described below. The advantage of perturbation method is that the governing equations and boundary conditions can be solved analytically and approximately. The flow is driven by axial DC electric field of strength E_0 and pressure gradient. According to the EDL theory, the relationship between the electric potential $\psi(R,\theta)$ and the net volumetric charge density $\rho_e(R,\theta)$ can be described by Poisson equation

$$\nabla^2\psi = -\frac{\rho_e}{\varepsilon}. \tag{2}$$

Based on the assumption of local thermodynamic equilibrium [41], the net volumetric charge density ρ_e of symmetric electrolyte is expressed as

$$\rho_e = -2n_0 z_v e \sinh\frac{z_v e\psi}{k_b T}, \tag{3}$$

where ε is the dielectric constant of the electrolyte liquid, z_v is the valence, n_0 is the ion density of bulk liquid, k_b is the Boltzmann constant, e is the electron charge, and T is absolute temperature.

The term $\sinh(z_v e\psi/(k_b T))$ can be approximated by $z_v e\psi/(k_b T)$ when ψ is small enough. This linearization is known as the Debye–Hückel approximation. Substituting this approximation to the Poisson–Boltzmann equation, Eq. (2) can be written as

$$\nabla^2\psi = \kappa^2\psi, \tag{4}$$

where $\kappa = z_v e(2n_0/\varepsilon k_b T)^{1/2}$ is the Debye–Hückel parameter and $1/\kappa$ represents the thickness of EDL. The relevant boundary conditions are given as

$$\psi(R,\theta) = \zeta \ \text{at} \ R = R_w, \tag{5a}$$

$$\frac{\partial\psi(R,\theta)}{\partial R} = 0 \ \text{at} \ R = 0, \tag{5b}$$

where the zeta potential ζ on the wall is constant [35].

The continuity, momentum balance equations can be expressed in the dimensional form as

$$\nabla \cdot \vec{U} = 0, \tag{6}$$

$$\rho\left(\frac{\partial \vec{U}}{\partial T} + \left(\vec{U}\cdot\nabla\right)\vec{U}\right) = -\nabla P + \mu\nabla^2\vec{U} + \rho_e(R,\theta)E_0\vec{e}_Z, \tag{7}$$

where P is the pressure of the liquid, μ is the dynamic viscosity. The boundary conditions at the wall surface and the centerline of channel are given as follows:

$$\vec{U}(R,\theta) = 0 \, \text{at} \, R = R_w, \tag{8a}$$