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Multiple slip effects on bioconvection of nanofluid flow containing gyrotactic microorganisms and nanoparticles

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ABSTRACT

In the present paper we attempt to formulate the equation of motion for bio convective flow of nanofluid containing gyrotactic microorganisms past a vertical stretching sheet. In the model we incorporate the multiple slip conditions at the boundary surface. Then by introducing a similarity transform we convert the governing boundary layer equations into nonlinear ordinary differential equations coupled with appropriate boundary conditions and solve it numerically by using Maple 17 software. The effects of different slip parameters on flow characteristics are surveyed through proper graphs and charts. After analysis it is found that slip parameters are highly correlated with various physical properties of the flow field. Also from numerically compute data it is detected that growth in nanoparticle density slip parameter consequences 23.64% shrinkage in mass transfer proportion and 10.11% decline in microorganism density number and alternatively heat movement speed lessened by 11.16% with the institution of temperature slip effect in the stream.

1. Introduction

From last few decades researchers had great interest on nanofluid as presence of nanoparticle in regular fluid can enhance the thermal conductivity of the base fluid at a significant label. It is used as an industrial tool to increase the heat transfer rate of fluids like water, ethanol, oil, toluene, etc. Any practical field where heat transfer or cooling or drug delivery is required has an efficient usefulness of nanofluid. That's why recently many researchers have investigated different properties of nanofluid. The boundary layer flow due to stretching sheet has many industrial and engineering applications in recent years. Crane [1] investigated the MHD flow over a stretching sheet in two dimensions. Gupta and Gupta [2] examined unsteady boundary layer flow over stretching sheet in presence of suction or blowing. Nazar et al. [3] studied the characteristics of boundary layer flow of rotating nanofluid. Nanofluid flow past a stretching sheet was investigated by Khan and Pop [4]. Vajravelu et al. [5] discussed heat transfer in the Ag-water and Cu-water nanofluids flow. Hamad et al. [6] investigated the magnetic effect on free convective flow of nanofluid. Ibrahim et al. [7] examined stagnation flow of nanofluids over a stretching sheet. Unsteady flow of nanofluid past a stretching/shrinking sheet was examined by Malvandi et al. [8]. A numerical study on MHD boundary layer flow of nanofluids over a nonlinear stretching surface was discussed by Mabood et al. [9]. Sandeep et al. [10] studied nanofluid flow over an exponentially stretching sheet. Khan et al. [11] recently examined the stagnation point nanofluid flow of variable viscosity over a stretching surface. In recent times Makinde et al. [12] scrutinizes nanofluid movement over an extending surface with inconstant viscosity and heat emission. Mabood et al. [13] surveys non-Darcian flow over a stretching area and deliberated Soret influence of the current. Also Mabood and Das [14] premeditated hydromagnetic flow over a stretching sheet bearing in mind second order slip.

Microorganism particles have extensively used to produce industrial and commercial products like ethanol, bio fuel made from waste, fertilizers etc. They are also used in water treatment plants. Hydrogen gas and biodiesel, a promising renewable energy source are produced by those microorganisms. That's why, we need to study the swimming patterns and mass transfer property of the microorganisms so that application of the organisms could be made more effective, profitable and extensive for the well being of the mankind. Bioconvection is formation of different types of random fluid patterns at the microscopic label due to the spontaneous swimming of self-propelled microorganisms which are present in the water and in the other liquids denser than water. The swimming of those microorganisms is biased by the natural phenomena like searching for nutrient, oxygen for respiration, optimizing light intake for photosynthesis. Hillesdon et al. [15] studied the theory of oxytactic bioconvection. Metcalfe and Pedley [16,17] examined bioconvection further and obtained non-linear theory for different flow patterns. Becker et al. [18] established a numerical modeling of bioconvection through a porous medium. The theory of bio-thermal convection due to temperature gradient and swimming of the microorganisms was introduced by Kuznetsov [19]. He also developed the theory of thermo bioconvection in porous medium [20]. Kuznetsov and

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Nield [21] studied natural convective flow for nanofluid. Das et al. [22] studied the nature of bioconvection in presence of chemical reaction. Raju et al. [23] investigated the bioconvection of non-Newtonian fluid past a rotating cone or plate. Mabood and Khan [24] analytically discussed the free convective flow of non-Newtonian fluids with Micro-organisms in porous media. In another paper Mabood et al. [25] depicts the thermal radiation effect of Casson fluid in circular cylinder.

When a fluid flow over a rigid surface the fluid does not penetrate into the surface or leave an empty space between the wall and fluid, the tangential and normal component of the velocity of the wall and the fluid are the same. Therefore at the boundary surface the velocity of the fluids is same as the velocity of the wall. It can be assumed that the molecules of the fluid and surface entangled at the boundary. But this is too simple view to be real at the molecular scale. The no-slip condition at the molecular scale need not be accurate in the continuum model whose scale is much larger. There are many situations where slip conditions must be considered such as micro valve, hard disk drive, micro nozzle, micro pumps etc. Micro-scale fluid dynamics in MEMS is applied various emerging industrial research interests in recent years. Under micro-scale dimensions, the fluid motion still follows the Navier–Stokes equations but under slip velocity boundary conditions. On the other hand, no-slip conditions are not appropriate for non-Newtonian fluids and for fluids with nanoparticles and microorganisms. Anderson [26] discussed the slip effect on the fluid flow over a stretching sheet. Wang [27] discussed about stagnation slip flow over a moving plate. Hayat et al. [28] introduced the thermal slip conditions in the viscous flow over a stretching sheet. Fang et al. [29] and Mahantesh et al. [30] discussed heat and mass transfer of boundary layer flow with second order slip conditions. Turkyilmazoglu [31] had found exact analytical solution of MHD nanofluid flow with slip condition. Aminreza et al. [32] tried to find partial slip effect on nanofluid flow with constant surface temperature numerically. Das [33] also examined numerically the nanofluid flow characteristics passing over a permeable sheet with surface slip. Ibrahim and Shankar [34] discussed MHD nanofluid flow considering velocity, thermal and solutal slip in the boundary conditions. Khan et al. [35] discussed velocity slip condition in nanofluid flow containing gyrotactic organisms. Mabood et al. [36] investigated casson fluid flow with multiple slip in the boundary conditions using Homotopy Analysis Method. In recent times Usman et al. [37] investigates the influence of slip effect and thermal radiation on heat and mass flow of a fluid current in porous medium.

Magnetohydrodynamic nanofluid flow containing gyrotactic microorganisms over a permeable stretching surface is studied under multiple slip conditions. In the presence of nanoparticles due to Brownian diffusion and thermophoresis the velocity slip, thermal slip and concentration slip has been observed at the microscopic label at the wall. Also for the random movement of the microorganisms their density slip also has been noticed at the wall. In the present work we consider the regular fluid with microorganisms and nanoparticle which is very much different with fluid in all aspects. That's why we consider the present flow under the multiple slip condition. We discuss the nature of different properties of the fluid with the change in various slip parameters with the help of correlation coefficients, graphs and Charts numerically and try to establish a relation between bioconvection and slip effect on the fluid.

2. Mathematical modeling of the flow

If we consider the swimming of the microorganisms, water is the natural choice for the base fluid. So in the present paper we develop a mathematical model of bioconvection in a water based steady incompressible nanofluid flow containing microorganisms under some assumptions (see Fig. 1). We consider the nanoparticle concentration in the base fluid to be less than 1% so that the presence nanoparticles should not affect the swimming directions and velocity of the microorganisms in the flow. The suspended nanoparticles stay stable in the fluid and do not agglomerate. As the velocity of the nanoparticle and

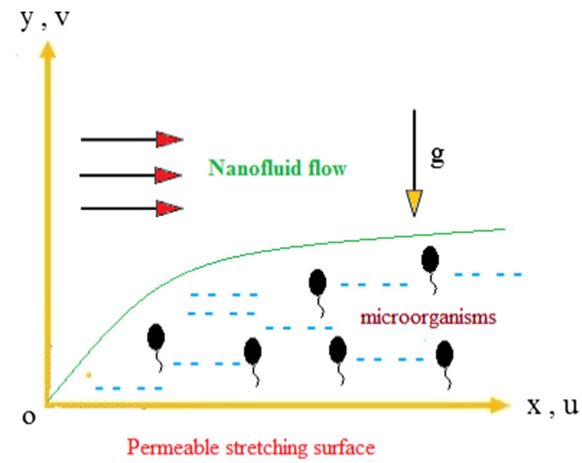

Fig 1. Physical model of the flow.

microorganisms increased, bioconvection could be instable. The fluid is to be diluted to avoid suspension of the nanoparticle. Since most of the microorganisms swim at very low Reynolds's number, we can neglect the effect of non- Newtonian stress and flow around a single cell as the cell size is very low compared to the large scale of fluid medium. Considering all those assumptions the following equations of conservation of total mass, momentum, thermal energy, nanoparticle concentration, microorganism density are formulated.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\begin{aligned}\frac{\partial p}{\partial x}=\mu\frac{\partial^2 u}{\partial y^2}-\sigma B_0^2 u-\rho_f\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-\frac{\varepsilon\mu}{k}u \\ +g\left[\left(1-\phi_\infty\right)\rho_{f\infty}\beta\left(T-T_\infty\right)-\left(\rho_p-\rho_{f\infty}\right)\left(C-C_\infty\right)-n\gamma\left(\rho_{m\infty}-\rho_{f\infty}\right)\right]\end{aligned} \tag{2}$$

$$\frac{\partial p}{\partial y}=0 \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{5}$$

$$u\frac{\partial n}{\partial x}+v\frac{\partial n}{\partial y}+\frac{bW_c}{(C_w-C_\infty)}\frac{\partial}{\partial y}\left(n\frac{\partial C}{\partial y}\right)=D_m\frac{\partial^2 n}{\partial y^2} \tag{6}$$

where u, v are the velocity components along the x and y-axis respectively, T stands for temperature, C represents nanoparticle concentration, n is the concentration of microorganisms, ε stands for the porosity and k represents the permeability of the porous medium, σ implies the electrical conductivity, B_0 is the magnitude of the applied transverse magnetic field, α is the thermal diffusivity of the nanofluid, β is the volumetric thermal expansion coefficient of base fluid, γ is the average volume of a microorganism, $\tau=(\rho c)_p/(\rho c)_f$ is the ratio of the effective heat capacity of the nanoparticle material and the base fluid, ρ_f is the density of base fluid, ρ_p is the nanoparticle density, ρ_m is the density of microorganisms, D_B represents the Brownian motion coefficient, D_T is the thermophoretic diffusion coefficient, D_m is the diffusivity of microorganisms, b is the chemotaxis constant and W_c is the maximum cell swimming speed, the subscript ∞ denotes the corresponding values at far field, the subscript w denotes the corresponding values at wall. We apply Oberbeck-Boussinesq approximation to linearize the buoyancy