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Three-dimensional flow and heat transfer to burgers fluid using Cattaneo-Christov heat flux model

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ABSTRACT

The aim of this communication is to explore the steady three-dimensional boundary layer flow and heat transfer characteristics to Burgers fluid utilizing Cattaneo-Christov heat flux model. This model is the generalization of classical Fourier's law that considers the fascinating aspect of thermal relaxation time. The governing boundary layer equations of motion and energy are reduced to a set of three ordinary differential equations by implementation of suitable transformations which are then solved analytically by utilizing the homotpy analysis method (HAM). The effects of the thermal relaxation time β and the ratio of stretching rates parameter α on the temperature field is analyzed and presented graphically. It is observed that the temperature distribution is significantly affected with varying values of the thermal relaxation time β.

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1. Introduction

A standout amongst the best models in continuum physics is the classical Fourier's [1] heat conduction it is utilized for the description of heat transfer mechanism in different correlated circumstances. However, it has one of major limitation that it yields a parabolic energy equation for temperature field and consequently it contradicts with the principle of causality. To conquer this difficulty Cattaneo [2] added a thermal relaxation time in the classical Fourier's-law of heat conduction which allows the transport of heat via propagation of thermal waves with finite speed. Such sort of heat transportation has exciting practical applications that span from nanofluid flows to the modeling of skin burn injury. After that Christov [3] modified the Cattaneo law by the time derivative in Maxwell-Cattaneo's model with the Oldroyd's upper-convected derivative in order to preserve the material-invariant formulation. Several authors including Straughan [4] studied Cattaneo-Christov model with thermal convection. Ciarletta and Straughan [5] analyzed the uniqueness of the solutions for the Cattaneo-Christov equations. Tibullo and Zampoli [6] provided the uniqueness of Cattaneo-Christov heat flux model for flow of incompressible fluids. Han et al. [7] investigated boundary layer stretched flow of a Maxwell fluid with Cattaneo-Christov heat flux model. Impact of Cattaneo-Christov heat flux in MHD flow of Oldroyd-B fluid with homogeneous-heterogeneous reactions is studied by Hayat et al. [8] Effectiveness of Cattaneo–Christov heat flux model in flow of variable thermal conductivity fluid over a variable thicked surface is addressed by Hayat et al. [9]. Abbasi and Shehzad [10] examined the heat transfer analysis for three-dimensional flow of Maxwell fluid with Cattaneo-Christov heat flux model. Hayat et al. [11] investigated the stagnation point flow with Cattaneo-Christov heat flux and homogeneous-heterogeneous reactions. Waqas et al. [12] addressed the impact of Cattaneo-

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Christov heat flux model for flow of variable thermal conductivity generalized Burgers fluid. Hayat et al. [13] discussed the impact of Cattaneo-Christov heat flux model on Jeffrey fluid flow with homogeneous-heterogeneous reaction. Hayat et al. [14] reported the three-dimensional rotating flow of Jeffrey fluid for Cattaneo-Christov heat flux model.

There is consistently increasing enthusiasm of current researchers in flows of non-Newtonian fluids because of their broad applications in industry and engineering. However, the fluid motion of non-Newtonian fluids is much more complicated and subtle in comparison with that of the Newtonian fluids. The non-Newtonian fluids are divided into three categories namely; differential, rate, and integral types. Amongst the non-Newtonian fluids, the rate type fluids are those which take into account the elastic and memory effects. The Burgers fluid model is a subclass of rate type fluids. This model has been successfully used to describe the motion of the earth's mantle. The Burgers model is the preferred model to depict the response of asphalt and asphalt concrete [15] as well as used to model the propagation of seismic waves in the interior of the earth [16]. Some fascinating studies involving Burgers fluid can be found in Refs. [17–23].

The primary goal of present analysis is to explore the rheological characteristics of three-dimensional boundary layer flow of Burgers fluid using Cattaneo-Christov heat flux model due to a bidirectional stretching sheet. The arising two-point boundary value problem treated analytically by the homotopy analysis method (HAM) [24–27] suggested by Liao [28]. Moreover, the effects of various controlling parameters are analyzed graphically and discussed in details.

2. Constitutive expression and equations

Consider the steady three-dimensional forced convection boundary layer flow of Burgers fluid over a bidirectional stretching surface. The sheet coincides with the plane $z=0$ and the flow takes place in the domain $z>0$. Heat transfer analysis is taken into account in the presence of Cattaneo-Christov heat flux model. The ambient fluid temperature is taken T_∞, while the surface temperature is maintained at a certain value of T_w such that $T_w>T_\infty$. The governing equations for flow and heat transfer of Burgers fluid are as follows:

$$div\mathbf{V}=0, \tag{1}$$

$$\rho(\mathbf{V}\cdot\nabla)\mathbf{V}=-\nabla p+\nabla\cdot\mathbf{S}, \tag{2}$$

$$\rho c_p(\mathbf{V}\cdot\nabla)T=\nabla\cdot\mathbf{q}, \tag{3}$$

$$\left(1+\lambda_1\frac{D}{Dt}+\lambda_2\frac{D^2}{Dt^2}\right)\mathbf{S}=\mu\left(1+\lambda_3\frac{D}{Dt}\right)\mathbf{A}_1, \tag{4}$$

$$\mathbf{q}+\lambda\left(\frac{\partial\mathbf{q}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{q}-\mathbf{q}\cdot\nabla\mathbf{V}+(\nabla\cdot\mathbf{V})\mathbf{q}\right)=-k\nabla T, \tag{5}$$

in which $\mathbf{V}$ denotes the velocity vector, T the temperature of the fluid, ρ the fluid density, p the pressure, c_p the specific heat of fluid at constant temperature. Furthermore, $\mathbf{S}$ the extra stress tensor, $\mathbf{q}$ the heat flux, $\mathbf{A}_1=(\nabla\mathbf{V})+(\nabla\mathbf{V})^{\mathrm{T}}$ the first Rivlin-Ericksen tensor, μ the dynamic viscosity, λ_1 and λ_3 $(\le\lambda_1)$ the relaxation and retardation times, respectively, λ_2 $(<\lambda_1\lambda_3)$ the material parameter of the Burgers fluid, k the thermal conductivity of the fluid, λ the thermal relaxation time and $\frac{D}{Dt}$

$$\frac{Da_i}{Dt}=\frac{\partial a_i}{\partial t}+u_r a_{i,r}-u_{i,r}a_i. \tag{6}$$

For a three-dimensional flow, with velocity $\mathbf{V}=[u(x,y,z),v(x,y,z),w(x,y,z)]$, temperature $T=T(x,y,z)$ and stress $\mathbf{S}=\mathbf{S}(x,y,z)$ fields, we obtain the following boundary layer equations. By utilizing the standard boundary layer approximations Eqs. (1)-(5) give

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{7}$$

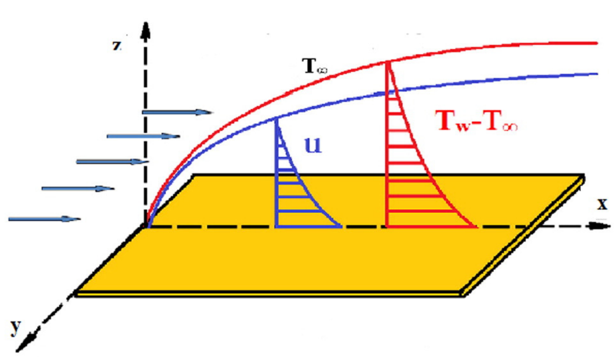

Fig. 1. Geometry of the problem.