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Role of slip velocity on peristaltic transport of couple stress fluid through an asymmetric non-uniform channel: Application to digestive system

G.C. Shit *, N.K. Ranjit

Department of Mathematics, Jadavpur University, Kolkata 700032, India

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ABSTRACT

Of concern in this paper is an investigation of peristaltic transport of a couple stress fluid in an asymmetric and non-uniform channel under the action of an externally applied magnetic field. The study is motivated towards investigating the peristaltic mechanism in the digestive system by considering the particle size effects. The effects of slip velocity on the channel walls have been taken into account. The nonlinearity of the problem is analyzed by using the long wave length and low Reynolds number approximations. The mathematical expressions for axial velocity, stream function, pressure gradient and pressure rise per wave length have been derived analytically. The above said quantities are computed for a specific set of values of the different parameters involved in the present model. The results estimated on the basis of the analytical solutions are presented graphically. The results presented here, show that the flow is appreciably influenced by the presence of slip velocity, couple stress parameter, magnetic field strength as well as the non-uniformity of the channel. This study puts forward an important observation that the occurrence of trapping bolus can be eliminated with suitably adjusting couple stress effect. Moreover, the slip velocity plays a significant role in trapping and transporting food bolus quickly in the digestive system.

1. Introduction

Peristaltic transport is a mechanism for transport of fluids induced by a progressive wave of area contraction or expansion along the length of a distensible tube. The peristaltic transport of viscous liquids is the most important biomechanical instrument of the digestive system. In physiology, peristalsis is used by the body to propel or mix the contents of a tube as in ureter, swallowing food through the esophagus, movement of chyme in the gastrointestinal tract, movement of ovum in the fallopian tube, vasomotion of small blood vessels, motion of spermatozoa in cervical canal, transport of bile in bile duct. Some worms use peristaltic mechanism for their locomotion. Roller and finger pumps also operate on the basis of this mechanism. The mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, blood pumps in heart lung machine, and transport of corrosive fluids where the contact of the fluid with the machinery parts is prohibited. It is also speculated that peristalsis may be involved in the translocation of water in tall trees. The translocation of water involves its motion through the porous matrix of the trees. The peristaltic transport of a toxic liquid is used in nuclear industry so as not to contaminate the outside environment. In MRI, the magnetic field is applied to view the bile ducts that scan internal tissues and organs. Moreover, the magnetic field can be used for drug delivery system using magnetic nanoparticles as carriers. Latham [1] who first introduced the motion of fluids in the peristaltic pump. Later on, the problem of mechanism of peristaltic transport has been rigorously studied by many investigators (Shapiro et al. [2], Fung and Yih [3], Jaffrin and Shapiro [4], Raju and Devanathan [5], Srivastava and Srivastava [6], Lozano [7], Misra et al. [8]) for better understanding of the mechanism of peristaltic transport. Abbas et al. [9] studied on the three-dimensional peristaltic flow of hyperbolic tangent fluid in a non-uniform channel having flexible walls and solved the problem by demonstrating homotopy perturbation method. Weinberg et al. [10] carried out experimental investigations of a two-dimensional peristaltic flow induced by sinusoidal waves. Their work was concerned with the measurements of mean flow, pressure gradient and pressure rise under low Reynolds number approximations.

The couple stress fluid may be considered as a special case of non-Newtonian fluid which is intended to take into account the particle size effects. The theory of couple stress fluid was first developed by Stokes [11] and represented the generalization of classical theory which allows for polar effects such as the presence of couple stress and body couples. Moreover, the couple stress fluid model is one of the numerous models that proposed to describe the characteristics of non-Newtonian fluids. The constitutive equations in this fluid model is very complex in which the order of differential equation is higher than the Navier-Stokes equations. Srivastava [12] studied the peristaltic

* Corresponding author.
E-mail addresses: gcs@math.jdvu.ac.in, gopal_iitkgp@yahoo.co.in (G.C. Shit).

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Nomenclature

(X', Y')	cartesian coordinates in fixed frame [m]
(U', V')	velocity components in fixed frame [m/s]
(x', y')	cartesian coordinates in wave frame [m]
(u', v')	velocity components in wave frame [m/s]
c	wave speed [m/s]
Q'	volumetric flow rate in fixed frame [m^3/s]
q	volumetric flow rate in wave frame [m^3/s]
p'	fluid pressure [Pa]
B_0	magnetic field strength [Wb/m^2]
R_e	Reynolds number ($\frac{cd_1\rho}{\mu}$)
H_a	Hartmann number ($B_0 d_1 \sqrt{\frac{\sigma}{\mu}}$)
$\vec{E}$	electric field [V/m]
$\vec{J}'$	current density [m^2/A]
F	dimensionless flow rate in wave frame
k	non-uniform parameter of the channel

Greek symbols

μ	co-efficient of viscosity [Pa·s]
γ	couple stress parameter ($d_1\sqrt{\frac{\mu}{\eta}}$)
λ	wave length [m]
ϕ	phase difference
σ	electrical conductivity [S/m]
α	inclination of the channel
ρ	density of fluid [kg/m^3]
δ	wave number ($\frac{d_1}{\lambda}$)
β	velocity-slip parameter
ψ	non-dimensional stream function
θ	dimensionless flow rate in fixed frame

transport of couple stress fluids in a tube by applying low Reynolds number and long wavelength approximations. He found that pressure rise is greater for a couple stress fluid than a Newtonian fluid model under similar circumstances. Elshehawey and Mekheimer [13] investigated the peristaltic flow of a couple stress fluid in a two dimensional channel by applying perturbation technique. They found that the mean axial velocity decreases with the couple-stress parameter, whereas it increases with the Reynolds number. Mekheimer [14] further studied on the peristaltic transport of a couple stress fluid in the uniform and non-uniform channels. It was found that the pressure rise for uniform channel is much smaller than that of the non-uniform channel. Ali et al. [15] extended the above problem to an asymmetric channel. They observed that the size of trapped bolus decreases with increasing couple stress parameter. Bhatti et al. [16,17] have examined the effects of slip-velocity and applied magnetic field on the peristaltic flow of Jeffrey fluid model through a two-dimensional non-uniform channel with porous medium. They observed that the variation of axial velocity reversed near the walls due to the presence of slip-velocity and magnetic field. Shit and Roy [18] have developed a mathematical model for the peristaltic motion of a micropolar fluid in a non-uniform symmetric porous channel in the presence of slip velocity and magnetic field. In a separate study, Shit and Roy [19] examined the hydromagnetic effect on inclined peristaltic flow of couple stress fluid in a uniform symmetric channel. Pandey and Choubey [20] investigated the effect of magnetic field on peristaltic transport of a couple stress fluid through a porous medium. Shit et al. [21] studied the effect of induced magnetic field on the peristaltic transport in an asymmetric channel by considering micropolar fluid model. Their study reveals that the consideration of induced magnetic field has significant impact on velocity profile, pressure rise per wave length as well as on streamlines.

It is well known that, at a microscopic level, the boundary condition for a viscous fluid at a rigid wall is "no-slip". However, in the case of physiological fluid, there might be possibility of existence of slip velocity at the flexible channel wall due to the presence of red cells slip [22]. Experimental study of Craig et al. [23] found that the slip length is depend on the viscosity and velocity of the fluid. In other words, for one dimensional flow, the velocity at the wall with slip length (β) can be expressed in terms of the velocity derivative normal to the wall as $u_s = \beta(\frac{\partial u}{\partial y})_{wall}$. Beavers and Joseph [24] proposed the slip condition in terms of the tangential components of the velocity and the stress at the boundary. Bhatti et al. [25] examined the slip effects and endoscopy analysis on blood flow induced by peristaltic wave through a non-uniform annulus by taking onto consideration of particulate fluid suspension model. They observed that the non-Newtonian behaviour has a decreasing effect on the pressure rise by means of the increasing particle volume fraction. Abbas et al. [26] and Rashidi et al. [27] have estimated the entropy generation in the flow of nanofluids through a channel with compliant walls induced by peristaltic wave having applications in blood flow. Ali et al. [28] investigated the peristaltic flow of a third grade fluid in a circular cylindrical tube when the no-slip condition at the tube wall is no longer valid. Ali et al. [29] also examined the peristalsis of a magnetohydrodynamic fluid with variable viscosity and slip velocity in a symmetric and uniform channel. Hayat et al. [30] studied the effects of slip velocity on peristaltic transport of a Maxwell fluid with heat and mass transfer phenomena. They pointed out that the slippage appears under a large tangential traction in the case of non-Newtonian fluid. Ellahi et al. [31] investigated the effects of heat and mass transfer on the peristaltic flow in a non-uniform rectangular duct with the assumptions of long wave length and low Reynolds number. Abbas et al. [32] carried out peristaltically magnetohydrodynamic flow of electrically conducting nanofluid in a non-uniform channel having application in drug delivery system. Interaction of couple stresses and velocity slip on peristaltic transport in uniform and non-uniform symmetric channel studied analytically by Sobh [33] without considering the effect of magnetic field. However, most of the above studies are restricted in a symmetric and uniform channel. The influences of Hall current and slip condition on the MHD flow induced by sinusoidal peristaltic wavy wall in two dimensional viscous fluid through a porous medium for moderately large Reynolds number have investigated by Mekheimer et al. [34], wherein they considered that the thickness of the boundary layer is larger than the amplitude of the wavy wall. Their study focuses on important applications for engineers that the fluid transportation takes place without an external pressure. Ellahi et al. [35] presented a theoretical study on peristaltic flow of Couple stress fluid in a symmetric non-uniform channel with compliant walls without examining the effects of magnetic field and slip boundary conditions. It has been proposed that boundary slip in aqueous systems is favored by hydrophobic surfaces in order to reduce the liquid-solid friction. Moreover, surface roughness and non-uniformity of the channel may also play a vital role in boundary slip. Therefore, our motivation is to examine the role of boundary slip when the non-uniform channel walls rhythmically contracting and expanding.

Owing to the abovementioned studies, we have investigated the effect of slip velocity on peristaltic flow of a couple stress fluid through an asymmetric and non-uniform channel. The long wave length and low Reynolds number assumptions have been made to simplify the highly nonlinear terms in the governing equations. The problem is solved analytically under the specified slip boundary conditions. Thus our study keep promise to give important result in the study of bio-fluid dynamics. As we know the real physiological problem is very complex, a theoretical analysis as just presented can only serve as a model which may help understanding of the peristaltic mechanism in the physiological system.

2. Mathematical modeling and analysis

Let us consider the peristaltic motion of a Couple stress fluid through an asymmetric and non-uniform two-dimensional channel (cf. Fig. 1).