Journal of Molecular Liquids 222 (2016) 101–108

Contents lists available at ScienceDirect

Journal of Molecular Liquids

journal homepage: www.elsevier.com/locate/molliq

Study of variable magnetic field on the peristaltic flow of Jeffrey fluid in a non-uniform rectangular duct having compliant walls

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ARTICLE INFO

Article history:
Received 23 May 2016
Received in revised form 18 June 2016
Accepted 3 July 2016
Available online 07 July 2016

Keywords:
Peristaltic flow
Jeffrey fluid
Variable magnetic field
Eigenfunction expansion method
Exact solutions

ABSTRACT

In this article, the effects of variable magnetic field on peristaltic flow of Jeffrey fluid in a non-uniform rectangular duct having compliant walls are investigated. The unsteady viscous incompressible electrically conducting flow is considered. The variable magnetic field under long wavelength and low Reynolds number is taken into account. The exact solutions of nonhomogeneous governing equations are obtained through eigenfunction expansion method. Impact of variables reflecting the salient features of magnetic parameter, Jeffrey parameter, aspect ratio, wall tension and wall properties has been graphically pointed out. Trapping phenomenon is analyzed through stream lines. A suitable comparison has also been made with the prior results in the literature as a limiting case of the considered problem, for instance, by taking a Jeffrey parameter $\lambda_1 = 0$, the presented result reduces to Newtonian fluid. It is worth mentioning that with the increment in magnetic field, it causes a reduction in the velocity of the fluid. Comparison with the existing published results are also presented as a special case of our study and found that presents results are in excellent agreement.

1. Introduction

Peristaltic transport of fluids in a channel/tube has acquired a special status among the recent investigators. Peristalsis is a wave like motion that occurs due to involuntary contraction and expansion of elastic wall. A large number of applications of peristaltic flow have been found in different fields of engineering and physiological processes. In bypass surgery it is used to circulate the blood in heart lung machine. This phenomenon naturally generated in the intestines, esophagus and stomach. The wave generated may have a short or long wave length. Peristaltic phenomena are the exposure of two important reflexes that are stimulated by a bolus of food stuff in the lumen. It is generally divided into two groups i.e., primary peristalsis and secondary peristalsis. Many engineers have invented different types of pumping machines such as roller pumps. Hariharan et al. [1] studied the peristaltic transport of non-Newtonian fluid in a diverging tube with different wave forms. Wang and Wu [2] have examined the unsteady flow of a fourth-grade fluid due to an oscillating plate. Ram and Takhar [3] analyzed the unsteady magnetohydrodynamic flow with a suspension of spherical particles through a square channel. Tripathi [4] studied a mathematical model for the peristaltic flow of chyme movement in the small intestine. To develop the understanding of the mechanism of peristalsis many authors have presented this phenomenon with different geometries. Some relevant studies on the topic can be seen from the list of references [5–13].

Moreover, the effects of magnetohydrodynamics (MHD) in peristalsis are very important and have great value in medical science such as magneto therapy, hyperthermia, arterial flow, compressor, optimization of blood pump machines, magnetic wound or cancer tumor treatment causing hyperthermia, bleeding reduction during surgeries and targeted transport of drug using magnetic particles as drug carries. As MHD has based on a fundamental law of electromagnetism that is when a magnetic field and an electric current intersect in a liquid, their repulsive intersection propels the liquid in a direction perpendicular to both the field and the current. Due to this fact, MHD is also used in the study of electrically conducting fluids; examples of such fluids include electrolytes, saltwater, liquid metals and plasmas. The controlled application of low intensity and frequency pulsating fields modify the cell and tissue. Magnetic susceptibility of chyme is satisfied the ions contained in the chyme or with heat generated by the magnetic field. The magnets could heat inflammations, ulceration, several diseases of uterus and bowel [14–19]. With all aforementioned points in mind, the peristaltic flows subject to different flow configurations and aspects have been analyzed, for instance, Abd Elmaboud [20] investigated the influence of the induced magnetic field on the peristaltic flow in an annulus. Mekheimer [21] studied the effect of the induced magnetic field on the peristaltic flow of a couple stress fluid. Eldabe et al. [22] have analyzed the peristaltically induced transport of a MHD by viscosity fluid in a non-uniform tube. Srinivas and Kothandapani [23] have

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http://dx.doi.org/10.1016/j.molliq.2016.07.013
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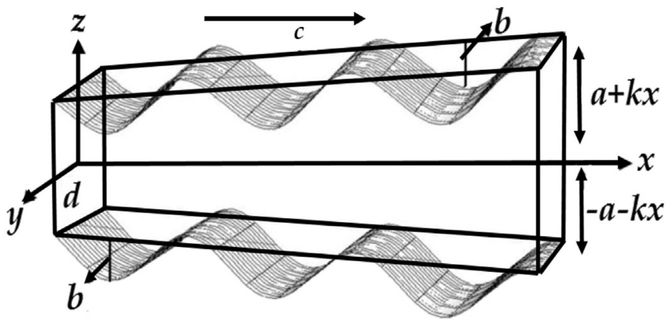

Fig. 1. Geometry of the problem.

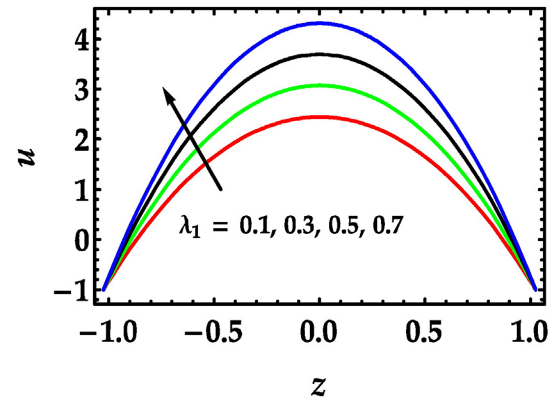

Fig. 3. Velocity profile for different values of λ_1.

investigated the influence of heat and mass transfer on MHD peristaltic flow through a porous space with compliant walls. From the literature survey it can be concluded that only a limited number of literature is available for peristaltic flow in rectangular channel. The first investigations of Reddy et al. [24] have examined the influence of lateral walls on peristaltic flow in a rectangular duct.

Furthermore, non-Newtonian fluids exhibit many applications [25–30] in bio-fluids that is why the physiological system has been examined by many investigators in order to find the treatment of diagnostic problems that arise during circulation in a human body. There are numerous models which have been proposed to describe such physiological fluids, however their full potential has not yet been exploited and a lot of questions remain unanswered. Among several models, non-Newtonian Jeffrey model is significant because Newtonian fluid model can be deduced from it as a special case by taking It is also speculated that the different physiological fluids such as blood, paint and toothpaste exhibit Newtonian and non-Newtonian behaviors.

The literature survey bears witness that the effects of space dependent magnetic field on peristaltic flow of Jeffrey fluid in a non-uniform rectangular duct having compliant walls has not yet been examined before. Keeping this fact in mind, the interest in exploring the effects for MHD peristaltic flow of non-Newtonian fluid in a non-uniform rectangular with compliant walls properties has been a motivating factor for this study. We in fact intend to strengthen our efforts to understand the problems of more complicated natures. This is particularly in modeling of peristaltic flow of non-Newtonian fluids in a duct having compliant walls, therefore, the constitutive relationships of Jeffrey fluid are used in the mathematical formulation. The exact solutions of nonlinear resulting equations are obtained. Comparison with the existing literature is also made. The graphs for different flow parameters of interest are sketched and analyzed. The trapping phenomena has also been discussed and displayed by drawing stream lines. It is worth to mention that the analytical results for the said model are presented here for the first time.

2. Mathematical formulation of the problem

Let us consider the flow of an incompressible Jeffrey fluid under the effects of variable magnetohydrodynamics in a non-uniform rectangular duct with wall compliances having a width $2d$ and height $2a+2kx$. The Cartesian coordinate system is chosen as shown in Fig. 1.

The flow is generated due to the imposition of the following sinusoidal waves on the compliant walls of channel

$$Z=H(x,t)=\pm a\pm kx\pm b\cos\left[\frac{2\pi}{\lambda}(x-ct)\right], \tag{1}$$

where, a is the height of the duct and b is the amplitudes of the waves, λ is the wavelength, c is the velocity of the propagation, t is the time and x is the direction of wave propagation. The walls parallel to xz-plane are not distracted and are not subject to any peristaltic wave motion. Let $(u,0,w)$ be the velocity for the flow in a rectangular duct then the governing equations for the incompressible Jeffrey fluid are stated as

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0, \tag{2}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}S_{xx}+\frac{\partial}{\partial y}S_{xy}+\frac{\partial}{\partial z}S_{xz}-\sigma B_0^2(y)u, \tag{3}$$

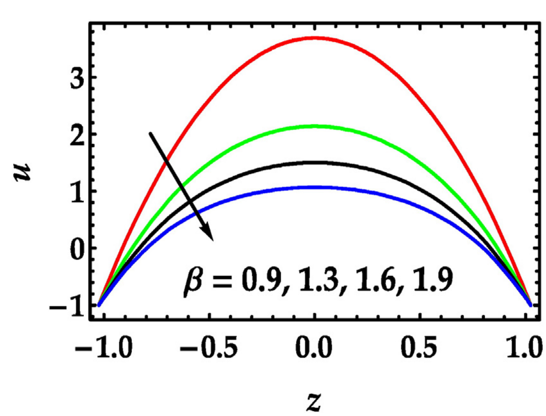

Fig. 2. Velocity profile for different values of β.

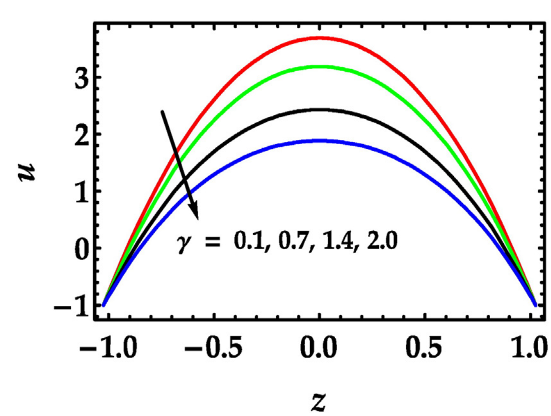

Fig. 4. Velocity profile for different values of γ.