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Influence of higher order chemical reaction and non-uniform heat source/sink on Casson fluid flow over a vertical cone and flat plate

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ABSTRACT

The present study focuses on the effects of surface dependent heat source/sink, viscosity and thermal conductivity variations on unsteady flow of Casson fluid over a vertical cone and flat plate with the influence of thermal radiation, temperature dependent heat source/sink and higher order chemical reaction effects. The model which is constituted with highly nonlinear governing equations is solved numerically by an efficient finite difference scheme of Crank–Nicolson type. Numerical calculations are carried out and reported graphically for various parametric conditions to show interesting aspects of the solution on flow, heat and mass transfer characteristics of this problem. Results indicate that the hydrodynamic and thermal boundary layer thickness decrease for higher values of thermal radiation parameter. The fluid flow and heat transfer are appreciably influenced by the temperature dependent heat source/sink parameter. The flow and heat transfer characteristics of this problem are also regulated by surface dependent heat source/sink, viscosity and thermal conductivity variation parameters.

1. Introduction

Casson fluid was introduced by Casson in 1959 to predict the flow behavior of pigment–oil suspensions of printing ink which exhibits yield stress in the constitutive equation. This fluid has a zero viscosity at an infinite rate of shear whereas it has an infinite viscosity at zero rate of shear. The Casson fluid model can be opted to fit the rheological data for several ingredients like jelly, tomato sauce, honey and soup. It is interesting to note that the Casson fluid model can also be preferred for human blood flow investigations because human blood has many constituents like protein, fibrinogen, red blood cells, etc. Some recent interesting contributions pertaining to the flow, heat and mass transfer characteristics of Casson fluids can be found in [14].

The thermal boundary layer is a thin layer around the body which describes the interactions of flow and thermal phenomena. Fluid temperature increases in the thermal boundary layer when a fluid flows around a hot body and the corresponding intensification in heat transfer result in proliferation of transport phenomena by diluting the fluid viscosity. Experimental data elucidates that the viscosity of liquids decreases across the momentum boundary layer when the fluid temperature increases whereas the reverse trend is observed in the case of gases. From the physical point of view, the kinematic viscosity should not be assumed as constant in flow region because the viscosity varies along with the rapid changes in fluid temperature. It is widely accepted that the consideration of fluid viscosity subject to temperature variation may improve the accuracy in modeling many fluid transport mechanisms encountered in hot rolling, wire drawing, glass fiber production, paper production, gluing of labels on hot bodies and drawing of plastic films [5–11].

The heat generated by the internal friction may affect thermal conductivity of the fluid. The consideration of thermal conductivity variations in heat transfer characteristics adds values to improve the heat transfer analysis of many engineering applications. Heat transfer in furnaces, boilers, porous burners, volumetric solar receivers, fibrous and foam insulations are some examples of combined conduction and radiation problems in which the changes of temperature and hence the variations of thermal conductivity are notable. Variable thermal conductivity in convective heat transfer problems may be observed in heat exchangers and cooling systems of electronic devices [12–18].

Thermal radiation plays a vital role in many engineering processes that operate at high temperature such as nuclear plants, gas turbines and various propulsion devices. In addition, thermal radiation plays a

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Nomenclature

u	component of dimensional velocity along x direction [ms^{-1}]
v	component of dimensional velocity along y direction [ms^{-1}]
r	dimensional local radius of the cone [m]
t^*	dimensional time [s]
g	gravitational acceleration [ms^{-2}]
k	variable thermal conductivity of the fluid [$Wm^{-1}\,K^{-1}$]
k_0	ambient thermal conductivity of the fluid [$Wm^{-1}\,K^{-1}$]
k_e	mean absorption coefficient [m^{-1}]
T	dimensional temperature of the fluid [K]
T_w	wall temperature [K]
T_∞	ambient temperature [K]
C_P	specific heat at constant pressure [$J\,kg^{-1}\,K^{-1}$]
C	dimensional concentration of the fluid [$mol\,m^{-3}$]
C_w	wall concentration [$molm^{-3}$]
C_∞	ambient concentration [$molm^{-3}$]
D	molecular diffusivity [m^2s^{-1}]
K_R	dimensional chemical reaction parameter
Gr_T	thermal Grashof number
N	buoyancy ratio parameter
P_r	Prandtl number
F	thermal radiation parameter
S_c	Schmidt number
K_r	chemical reaction parameter
l	order of chemical reaction (natural number)
u_p	dimensional velocity of the moving cone and plate [ms^{-1}]
Nu	local Nusselt number
Sh	local Sherwood number
$\overline{Nu}$	average Nusselt number
$\overline{Sh}$	average Sherwood number

Greek symbols

ρ	density [$kg\,m^{-3}$]
β	Casson fluid parameter
β_T	thermal expansion coefficient
β_C	concentration expansion coefficient
α	semi vertical angle of the cone
μ	dynamic viscosity [$kg\,m^{-1}\,s^{-1}$]
μ_0	ambient fluid viscosity at temperature T_∞
ν_0	ambient kinematic viscosity [$m^2\,s^{-1}$]
γ	viscosity variation parameter
δ	thermal conductivity variation parameter
σ_s	Stefan–Boltzmann constant [$Wm^{-2}K^{-4}$]
τ	local skin-friction
$\bar{\tau}$	average skin friction

significant role in the heat transfer characteristic of absorbing–emitting fluids when convection heat transfer is small, particularly in free convection problems. It is well known that the blood flow regulates the temperature of the human body and controls it according to the environment. Nowadays, the thermal regulation in human blood flow by means of thermal radiation is very important in several medical treatments for muscle spasm, myalgia, chronic wide-spread pain and permanent shortening of muscle. Moreover, it is equally important in many biomedical engineering and several thermal therapeutic procedures [19–25].

The heat transfer reflections measured in extensive surroundings differ from the results obtained in heat source/sink performance so that the heat source/sink is placed within compact enclosures to obtain effective control mechanisms on the heat transfer characteristics of various modern electronic products. This kind of heat source/sink is often employed to provide thermal control in some standard building blocks of electronic systems. Most of the archival literature on heat transfer studies projects the influence of temperature dependent heat source/sink only; the reason being that the temperature dependent heat source/sink is often exposed to control the heat and mass transfer characteristics of the fluid. In many situations, the surface dependent heat source/sink is regarded as an ineffectual heat transfer mechanism. However, some mathematical models are proposed to predict the behavior of heat transfer characteristics with the influence of surface dependent heat source/sink in addition to the temperature dependent heat source/sink which aids to improve the accuracy of heat transfer characteristics [26–31].

The study of heat generation/absorption effects in fluids is important in view of several physical problems such as fluids undergoing chemical reactions. Thus in many industrial processes involving flow, heat and mass transfer over a surface, the diffusing species can be generated or absorbed due to some kind of chemical reaction with the ambient fluid which can greatly affect the flow and mass transfer characteristics. Processes involving the mass transfer effect have been recognized as important mainly in chemical processing equipments. The species generation in a homogeneous reaction is analogous to internal source of heat generation. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. A chemical reaction is said to be of the first order and homogeneous if its rate is directly proportional to the concentration and it is a single-phase volume reaction. In addition, diffusing species generation or consumption caused by the reaction can be proportional to the l^{th} order concentration difference between the surface and ambient fluid where l is a natural number. Convective heat and mass transfer with chemical reaction plays an important role in meteorological phenomena, burning of haystacks, spray drying of milk, fluidized bed catalysis and cooling towers [32–33]. Some interesting contributions pertaining to the flow and heat and mass transfer characteristics can be found in Refs.[34–36].

The purpose of this investigation is to explore the influence of surface dependent heat source/sink, viscosity and thermal conductivity variations on unsteady flow of Casson fluid in the presence of thermal radiation, temperature dependent heat source/sink and higher order chemical reaction effects within the boundary layer over a vertical cone and flat plate by using the Crank–Nicolson method. The flow of Casson fluids (such as drilling muds, clay coatings and other suspensions, certain oils and greases, polymer melts and many emulsions), in the presence of heat transfer is an important research area due to its relevance in the optimized processing of chocolate, toffee, and other foodstuffs. Graphical results are presented and discussed for various physical parameters entering into the problem.

2. Mathematical formulation

A two dimensional, laminar, free convective flow of an incompressible Casson fluid over a vertical cone and a flat plate is considered. As seen from Fig. 1, x is the coordinate along the surface of the cone and plate measured from the origin and y is the coordinate normal to the surface. The cone and plate are placed with their axis of symmetry along the vertical direction and the origin of the coordinate system is placed at the vertex of the cone and plate. The wall $y = 0$ is maintained at constant temperature T_w and concentration C_w, higher than the ambient temperature T_∞ and ambient concentration C_∞, respectively. Casson's constitutive equation represents a nonlinear relationship between stress and rate of strain. The Casson fluid model has been found to be accurately applicable to analyze the characteristics of silicon suspensions, suspensions of bentonite in water and lithographic varnishes used for printing inks. In addition to that Casson fluid characteristics are typically found in fluids such as tomato sauce, honey, soup, and concentrated fruit juices. It is interesting to note that the Casson fluid model is