Journal of Molecular Liquids 219 (2016) 748–755

Contents lists available at ScienceDirect

Journal of Molecular Liquids

journal homepage: www.elsevier.com/locate/molliq

Heat transfer analysis of unsteady oblique stagnation point flow of elastico-viscous fluid due to sinusoidal wall temperature over an oscillating-stretching surface: A numerical approach

Abuzar Ghaffari [a,*], Tariq Javed [a], Kai-Long Hsiao [b]

[a] Department of Mathematics and Statistics, International Islamic University, Islamabad 44000, Pakistan
[b] Taiwan Shoufu University, 168, Nansh Li, Madou District, Madou Jen, Tainan, Taiwan, ROC

ARTICLE INFO

Article history:
Received 1 December 2015
Accepted 6 April 2016
Available online xxxx

Keywords:
Elastico-viscous fluid
Oblique stagnation point
Spectral method
Oscillating surface

ABSTRACT

In this article, heat transfer analysis of an unsteady oblique stagnation point flow of elastico-viscous Walter's B fluid over an oscillating-stretching surface, heated due to sinusoidal wall temperature is presented. The governing partial differential equations are transformed into dimensionless form. The solution of obtained partial differential equations is computed numerically using Chebyshev Spectral Newton Iterative Scheme (CSNIS). The computed results are highly accurate and compared with previous studies in limiting sense. The effects of involving parameters on the fluid flow and heat transfer are shown through tables and graphs. It is importantly noted that the amplitude of the local Nusselt number and skin friction coefficient enhances due to increase in the values of unsteady parameter. The heat transfer rate increases, with increase in the values of Prandtl number. In non-Newtonian fluid, the heat transfer rate decreases as compare to Newtonian fluid case. The variation of skin friction coefficient and local Nusselt number are discussed for the wide range of time and various pertinent parameters.

1. Introduction

Oblique stagnation-point flow appears when fluid from any source impinges obliquely on a rigid wall at an arbitrary angle of incidence as shown in Fig. 1. Many researchers have studied the steady two-dimensional oblique stagnation-point flow of Newtonian fluid. Stuart [1] was the first, who studied the orthogonal stagnation point flow. Later on, Tamada [2] and Dorrepaal [3] had extended the work of Stuart [1] by considering the oblique stagnation point flow. Reza and Gupta [5] generalized the problem of Chiam [4] by studying the oblique stagnation point flow over a stretching surface. In their paper, they ignored the displacement thickness and pressure gradient. This was partially rectified in a later study by Lok et al. [6]. Reza and Gupta [7] gave a correct solution to the above problem by fixing the errors in [5,6]. Drazin and Riley [8], Tooke and Blyth [9] reviewed the problem and included a free parameter associated with the shear flow component, which is related to the pressure gradient. Weidman and Putkaradze [10,11] studied the steady oblique stagnation-point flow impinging on a circular cylinder. Recently, Husain et al. [12], Mahapatra et al. [13], Lok et al. [14], Javed et al. [15,16], Hsiao [17], Zaheer et al. [18,19] have done notable work on orthogonal and oblique stagnation point flows.

Non-Newtonian fluids are very significant owing to their wide use in industries. The non-Newtonian fluid suspensions are usually encountered by civil, metallurgical, mining and chemical engineering. Many fluids in nature have very complex behavior and cannot be predicted from Newtonian fluid model. Experimentally different models are presented to predict the behavior of non-Newtonian fluids. Many viscoelastic fluid models [20–28] have been proposed but here constitutive equations of Walter-B fluid [29–31] are employed in the mathematical formulation.

In literature, different types of surface conditions have been employed, including constant surface heat flux, constant surface temperature, convective boundary conditions and recently the heating resulting from a catalytic surface reaction [32–37]. In all these studies, the constant surface conditions have been taken into account, which give a clear information about some basic process but it is not a realistic assumption. The values of surface temperature does not remains constant, it often fluctuate about some mean value. Following the Merkin [38], Merkin and Pop [39], Kelleher and Yang [40] and Brown and Riley [41], we have taken prescribed surface temperature as sinusoidal in the region of oblique station point flow of elastico-viscous fluid over an oscillating-stretching surface. The surface temperature oscillates about some mean value T_w and its value is greater than the ambient temperature T_∞ of the surrounding medium. An efficient numerical scheme namely Chebyshev Spectral Newton Iterative Scheme [42] (CSNIS) is implemented. The results are compared with the previous

* Corresponding author.
E-mail address: abuzar.iiui@gmail.com (A. Ghaffari).

http://dx.doi.org/10.1016/j.molliq.2016.04.014
0167-7322/

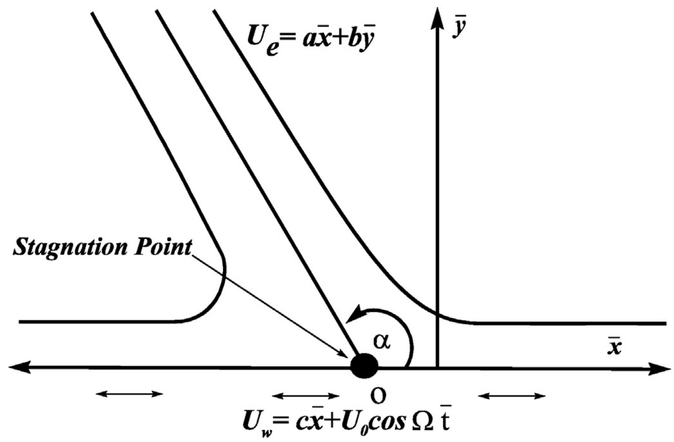

Fig. 1. Geometry of the unsteady two-dimensional elastico-viscous fluid flow which is impinging obliquely over an oscillating-stretching surface.

studies in limiting sense, which are in excellent agreement. The graphical results are interpreted with respect to various parameters of interest. The streamlines and isotherms are also plotted.

2. Problem formulation

We considered the unsteady two-dimensional flow of elastico-viscous Walter's B fluid impinging obliquely over an oscillating-stretching surface at $y = 0$ as shown in Fig. 1. The elasticity of the fluid is assumed constant throughout the flow regime. The temperature of surface is taken as sinusoidal, oscillating about the mean value T_w, which is higher than the ambient temperature T_∞ of the surroundings. The flow and energy equations are (see ref. [31] and ref. [39])

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0, \tag{1}$$

$$\begin{aligned}\frac{\partial \bar{u}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}}+\nu\nabla^2\bar{u}-\frac{k_0}{\rho}\Bigg[\frac{\partial}{\partial \bar{t}}\left(\nabla^2\bar{u}\right)+\left(\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right)\nabla^2\bar{u}\\ -\frac{\partial \bar{u}}{\partial \bar{x}}\nabla^2\bar{u}-\frac{\partial \bar{u}}{\partial \bar{y}}\nabla^2\bar{v}-2\left\{\frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2\bar{u}}{\partial \bar{x}^2}+\frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2\bar{u}}{\partial \bar{y}^2}+\left(\frac{\partial \bar{u}}{\partial \bar{y}}+\frac{\partial \bar{v}}{\partial \bar{x}}\right)\frac{\partial^2\bar{u}}{\partial \bar{x}\partial \bar{y}}\right\}\Bigg],\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial \bar{v}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}}=-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}}+\nu\nabla^2\bar{v}-\frac{k_0}{\rho}\\ \Bigg[\frac{\partial}{\partial \bar{t}}\left(\nabla^2\bar{v}\right)+\left(\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right)\nabla^2\bar{v}-\frac{\partial \bar{v}}{\partial \bar{x}}\nabla^2\bar{u}-\frac{\partial \bar{v}}{\partial \bar{y}}\nabla^2\bar{v}\\ -2\left\{\frac{\partial \bar{u}}{\partial \bar{x}}\frac{\partial^2\bar{v}}{\partial \bar{x}^2}+\frac{\partial \bar{v}}{\partial \bar{y}}\frac{\partial^2\bar{v}}{\partial \bar{y}^2}+\left(\frac{\partial \bar{u}}{\partial \bar{y}}+\frac{\partial \bar{v}}{\partial \bar{x}}\right)\frac{\partial^2\bar{v}}{\partial \bar{x}\partial \bar{y}}\right\}\Bigg],\end{aligned} \tag{3}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}}=\frac{k_f}{\rho C_p}\left(\frac{\partial^2\bar{T}}{\partial \bar{x}^2}+\frac{\partial^2\bar{T}}{\partial \bar{y}^2}\right). \tag{4}$$

In the above equations, $\bar{u}(\bar{x},\bar{y})$ and $\bar{v}(\bar{x},\bar{y})$ are the velocity components in $\bar{x}$ and $\bar{y}$-directions, $\bar{T}(\bar{x},\bar{y})$ is the temperature, $\bar{p}(\bar{x},\bar{y})$ is the pressure, ν is the kinematic viscosity, ρ is the density, k_0 is elasticity, C_p is the specific heat and k_f is thermal conductivity of the fluid. The boundary conditions of the problem can be defined as

$$\begin{aligned}&\bar{y}=0: \quad \bar{u}=c\bar{x}+U_0\cos\Omega\bar{t},\ \bar{v}=0,\ \bar{T}=T_\infty+\Delta T(1+\varepsilon_1\sin\Omega\bar{t}),\\ &\bar{y}\to\infty: \quad \bar{u}=a\bar{x}+b\bar{y},\ \bar{T}=T_\infty,\end{aligned} \tag{5}$$

where a, b and c are positive constant of dimension [1/T], T_∞ is the ambient temperature and $\Delta T = T_w - T_\infty$ is some temperature scale, ε_1 is the amplitude of the imposed temperature oscillation, Ω is the frequency of the oscillation. Upon using the following non-dimensional variable

$$x=\bar{x}\sqrt{\frac{c}{\nu}},\ \ y=\bar{y}\sqrt{\frac{c}{\nu}},\ \ t=\Omega\bar{t},\ \ u=\frac{1}{\sqrt{\nu c}}\bar{u},\ \ v=\frac{1}{\sqrt{\nu c}}\bar{v},\ \ p=\frac{1}{\rho\nu c}\bar{p},\ \ T=\frac{\bar{T}-T_\infty}{T_w-T_\infty} \tag{6}$$

in Eqs. (1–5), we get the following form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{7}$$

$$\begin{aligned}\frac{\Omega}{c}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\nu\nabla^2u-\frac{k_o c}{\rho\nu}\\ \left[\frac{\Omega}{c}\frac{\partial}{\partial t}\left(\nabla^2u\right)+\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\nabla^2u-\frac{\partial u}{\partial x}\nabla^2u-\frac{\partial u}{\partial y}\nabla^2v\right.\\ \left.-2\left\{\frac{\partial u}{\partial x}\frac{\partial^2u}{\partial x^2}+\frac{\partial v}{\partial y}\frac{\partial^2u}{\partial y^2}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\frac{\partial^2u}{\partial x\partial y}\right\}\right],\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\Omega}{c}\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\nu\nabla^2v-\frac{k_o c}{\rho\nu}\\ \left[\frac{\partial}{\partial t}\left(\nabla^2v\right)+\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\nabla^2v-\frac{\partial v}{\partial x}\nabla^2u-\frac{\partial v}{\partial y}\nabla^2v\right.\\ \left.-2\left\{\frac{\partial u}{\partial x}\frac{\partial^2v}{\partial x^2}+\frac{\partial v}{\partial y}\frac{\partial^2v}{\partial y^2}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\frac{\partial^2v}{\partial x\partial y}\right\}\right],\end{aligned} \tag{9}$$

$$\frac{\Omega}{c}\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_f}{\rho\nu C_p}\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}\right), \tag{10}$$

$$\begin{aligned}&y=0: \quad u=x+\varepsilon\cos t,\ v=0,\ T=1+\varepsilon_1\sin t,\\ &y\to\infty: \quad u=\frac{a}{c}x+\frac{b}{c}y,\ T=0.\end{aligned} \tag{11}$$

In which $\varepsilon = U_0/\sqrt{\nu c}$ is the dimensionless constant which describes the amplitude of the plate oscillation. Introducing the stream function ψ, which satisfies the continuity Eq. (7) identically, we write the velocity components as

$$u=\frac{\partial\psi}{\partial y},\ \ v=-\frac{\partial\psi}{\partial x}. \tag{12}$$

By eliminating pressure from Eqs. (8) and (9) and then after using Eq. (12), Eqs. (7–11) take the following form

$$\beta\frac{\partial\left(\nabla^2\psi\right)}{\partial t}+We\beta\frac{\partial\left(\nabla^4\psi\right)}{\partial t}-\frac{\partial\left(\psi,\nabla^2\psi\right)}{\partial(x,y)}-We\frac{\partial\left(\psi,\nabla^4\psi\right)}{\partial(x,y)}-\nabla^4\psi=0, \tag{13}$$

$$\beta\frac{\partial T}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{1}{\Pr}\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}\right), \tag{14}$$

$$\begin{aligned}&y=0: \frac{\partial\psi}{\partial y}=x+\varepsilon\cos t,\ \ \psi=0,\ \ T=1+\varepsilon_1\sin t,\\ &y\to\infty: \psi=\frac{a}{c}xy+\frac{\gamma}{2}y^2,\quad T=0,\end{aligned} \tag{15}$$

where $\gamma = b/c$ represents shear in the stream, $\beta = \Omega/c$ is dimensionless unsteady parameter, $We = k_0c/\rho\nu$ be the Wiessenberg number and $Pr = \mu C_p/k_f$ be the Prandtl number. Suppose the solution of Eqs. (13–14) subject to boundary conditions (Eq. 15) is of the form

$$\psi=xf(y)+g(y,t),\quad T=\theta(y,t), \tag{16}$$