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Water driven flow of carbon nanotubes in a rotating channel

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ABSTRACT

In this article, flow and heat transfer effects of both single and multiple wall carbon nanotubes within the base fluid (water) are analyzed between two rotating plates. Moreover, we have considered that the upper wall of the channel is permeable while the lower wall is moving with variable velocity to produce the forced convection along with the Coriolis and centripetal forces with the rotation of fluid. The compatible transformations have been used to construct the non-dimensional system of governing equations. Numerical simulation is performed to obtain the solutions structure. Thermophysical properties of each base fluid and nano particle are incorporated in the form of thermal conductivity, viscosity, density, specific heat, nanoparticle volume fraction and Prandtl number to attain the solution of the model. It is found that water based single wall carbon nanotubes (SWCNTs) produce less drag and high heat transfer rate as compared to the water based multiple wall carbon nanotubes (MWCNTs). Influence of rotation causes the drag increase and decreases the Nusselt number irrespective of the other pertinent parameters. Moreover suction/injection plays an important role in determining the peak position of velocity. The effect of suction/injection is shown through plotting streamlines.

1. Introduction

Fluid flow and heat transfer in a channel is one of the most important phenomena in high performance heat exchangers, chemical reactors, nuclear power plants, high temperature boiling plants, petroleum refineries and refrigeration process. During these processes the basic principle is to efficiently manage the heat transfer process and reduce the drag. Flows in rotating machines are modeled by using the rotating frame of references. Rotating flows have numerous applications in applications in turbomachinery. Rotating frames are not physically rotating anything and therefore transient effects are not visible due to the real motion. The rotating frame of reference approach can be used to solve the problems where transient effects due to rotor–stator interaction are small. A typical example is the mixing tank where the impeller–baffle interactions are relatively weak; large-scale transient effects are not present. The energy transfer between the fluid and rotor is an important feature in several rotating machines. Therefore, the study of rotating flows in the presence of nanoparticles will explore the possibility of heat transfer enhancement.

In heat transfer studies one field which has gained immense interest from the researchers is nanofluid flow. Nanofluids are a of base fluid and nanoparticles. Nanoparticles can be metallic (Cu, Zn, Al, etc.) or non-metallic and even chain of particles (carbon nanotubes). Nanofluids have considerable advantages over the ordinary fluids due to the enhanced thermal conductivity. Initially, Choi [1] introduced the concept of nanofluid based on incorporation of nano sized (10^{-9} to 10^{-11} nm) particles in a fluid. Masuda et al. [2] showed the thermal conductivity enhancement due to addition of ultra-fine sized particles, which were called nanoparticles. After Masuda [2], Pak and Cho [3] discussed the heat transfer due to addition of submicron metallic oxide particles in the base fluid. Their study concluded that the alteration in heat transfer rate was mainly due to higher thermal conductivity of submicron particles. Recently, several studies [4–6] were published on the analysis of heat transfer due to incorporation of nanoparticles in base fluid. Boungiorno [7] developed a mathematical model to study nanofluid flow. Seven slip phenomena were considered which can be important and proved that only Brownian motion and Thermophoresis are the dominant ones in relevance to nano fluids. This model hasn't taken into account the effect of shapes of nanoparticles in heat transfer. Recent studies [8–10] have showed that shape of and nature of nanoparticles are important factors to enhance the thermal conductivity of working fluid. Effect of shape on heat transfer has been examined by Elias et al. [11], they considered five different shape nanoparticles (cylindrical, spherical, bricks, blades and platelets). Their study showed that although all of them increase the heat transfer characteristics yet cylindrical shape nanoparticles have higher heat transfer rate. Vanaki et al. [12] discussed the effect of turbulent nanofluid flow in a wavy channel under different shaped SiO_2 nanoparticles. They concluded that platelet shape particles have better performance in heat transfer phenomenon. Jeong et al. [13] showed that particle shape has significant effect on the viscosity and thermal conductivity of base

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fluid. They found that rectangular shaped particles increased the viscosity of base fluid by 69% and it is 7.7% higher than that of the spherical shape particles. Timofeeva et al. [14] experimentally showed that shape of the nanoparticle played an important role in percentage increase in the thermal conductivity of base fluid and blade shaped nanoparticles have higher thermal conductivity compared to the rest of shapes.

Carbon nanotubes (CNTs) are cylindrical shapes like structures of carbon atoms with diameter ranges between 1 and 50 nm. They exhibit exceptional electrical, mechanical, thermal and optical properties at individual level. According to the details given by the CNT manufacturing firm NTI, at individual tube level CNT particles have; two hundred times strength and five times elasticity of steel, fifteen times thermal conductivity and thousand times the current capacity of copper and half the density of aluminum. Moreover CNTs don't possess any threat to the environment due to presences of carbon chains. Thus Environmental Protection Agency (EPA) declared them articles which are not hazardous or toxic for the environment. Hence, it is important to explore the effects of CNT on the fluid flow and heat transfer of Newtonian and non-Newtonian fluids. Haq et al. [15] considered the water based flow in the presence of single wall carbon nanotubes (SWCNTs) and multiple wall carbon nanotubes (MWCNTs). Their study reported the higher Nusselt number and skin friction for SWCNT than the MWCNT. In another study Haq et al. [16] showed that engine oil based CNT fluid has higher skin friction and heat transfer rate as compared to water and kerosene based CNT fluid.

Stretching is an important phenomenon in the production of polymer sheets, drawing of copper wires and film coatings. Sakiadis [17] pioneered the idea of boundary layer flow over a continuously moving surface and modeled the two-dimensional boundary layer equations. Tsou et al. [18] extended this work to examine the effects of stretching sheet on the momentum and heat transfer. Erickson et al. [19] took it further to examine the effect of mass transfer by considering suction/injection at the wall. Nadeem and Hussain [20] discussed the flow of Williamson nanofluid over a linearly stretching surface. It has been showed that Lewis number appearance in the governing equations depends upon what types of similarity transformations have been chosen. In another article, Hussain et al. [21] discussed the micro-rotation effect on the nanofluid flow over a linearly stretching surface. It has been showed that micro-rotation has a decreasing effect on skin friction while an increasing effect on heat transfer rate. In recent years, several studies [22–30] have been reported on the boundary layer flow of a nanofluid over a continuously moving surface. But only few articles have been written considering the nanofluid flow in a channel with the stretching of lower plate along the axis of flow. Borakati and Bharali [31] examined the effect of magnetic field on the two-dimensional channel flows with heat transfer in a rotating system. Hussnain et al. [32] obtained the HAM solution for three-dimensional flow of second grade in a rotating frame. Large values of rotation parameter caused the reversal of flow and fully develop flow is achieved with the increasing values of suction/injection parameter. Vajravelu and Kumar [33] presented the analytic and numerical solution for the two-dimensional flow in a channel and discussed the effect of rotation parameter on the velocity. Freidoonimehr [34] discussed the effects of different nanoparticles on the velocity and skin friction of the water based fluid in the rotating channel. Recently, Sheikholeslami et al. [35] discussed the Cu–water nanofluid flow between a porous plate and a stretching surface in a rotating system. Their study reported that the skin friction coefficient and the Nusselt number are greatly influenced by the presence of nanoparticles. Moreover the coefficient of skin friction and Nusselt number decreases with the increase in rotation parameter. Recent studies attain the considerable attention at industrial level and daily usage lubricant to transfer, remove or add the heat based on thermal conductivity [36–40].

The aim of the present article is to discuss the effects of single and multiple wall carbon nanotubes on the fluid flow and heat transfer of water based fluid in a rotating frame of reference where the lower plate is continuously moving along the horizontal axis and the upper plate is porous. Thermo-physical properties of SWCNTs, MWCNTs, Prandtl number and densities are used to develop the steady state solutions. Flow field results are discussed through plotting the graphs of velocity and temperature profile. Moreover the drag and heat transfer rate are examined by plotting the graphs of skin friction coefficient and Nusselt number against the nanoparticle volumetric fraction and pertinent parameters. To the best of the authors' knowledge no study reports the effect of CNTs on the fluid flow and heat transfer in the rotating channel.

2. Mathematical modeling

Let us consider an incompressible flow of water based CNTs confined between two infinite plates in a rotating frame of reference (angular velocity $\boldsymbol{\Omega}[0,\Omega,0]$). The upper plate is porous and the lower plate is moving with velocity $U_w = ax$ $(a>0)$. The steady state velocity field is defined as $\boldsymbol{V}[u(x,y),v(x,y),w(x,y)]$, where u, v, w are the velocity components along x, y and z direction, respectively. The schematic diagram of the problem is shown in Fig. 1:

The momentum governing equation for rotating flow is

$$\rho_{nf}\left(\frac{d\boldsymbol{V}}{dt}+2\boldsymbol{\Omega}\times\boldsymbol{V}+\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})\right)=div\boldsymbol{T}. \tag{1}$$

$2\boldsymbol{\Omega}\times\boldsymbol{V}$ is the Coriolis force whose direction is perpendicular to both $\boldsymbol{\Omega}$ and $\boldsymbol{V}$. $\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{r})$ is the centripetal force, which is also perpendicular to both $\boldsymbol{\Omega}$ and $\boldsymbol{V}$ but is directed toward the axis of rotation. $\boldsymbol{T}$ is the viscous stress tensor and ρ_{nf} is the density of nanofluid. In component form it can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+2\Omega\, w\right)=-\frac{\partial p^*}{\partial x}+\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{3}$$

$$\rho_{nf}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p^*}{\partial y}+\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{4}$$

$$\rho_{nf}\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}-2\Omega \mathrm{u}\right)=\mu_{nf}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right), \tag{5}$$

Where $p^*=p-\frac{\Omega^2x^2}{2}$ is the modified pressure and μ_{nf} is the effective dynamic viscosity of nanofluid, the absence of $\frac{\partial p^*}{\partial z}$ depicts the net cross flow along z-axis. The heat transfer phenomenon can be expressed mathematically as

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{(\rho c)_{nf}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{6}$$

T denotes the temperature of the fluid. Where $\alpha_{nf}=\frac{k_{nf}}{(\rho c)_{nf}}$ is the thermal diffusivity of the nanofluid i.e. the ratio of effective heat capacity $(\rho c)_{nf}$ of the nanofluid to effective thermal conductivity k_{nf} of nanofluid. The effective density of the nanofluid is given by

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_{CNT}, \tag{7}$$

The heat capacity of the nanofluid and the effective dynamic viscosity μ_{nf} are defined as

$$(\rho c)_{nf}=(1-\phi)(\rho c)_f+\phi(\rho c)_{CNT}, \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}, \tag{8}$$