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# Effect of magnetic dipole on viscous ferro-fluid past a stretching surface with thermal radiation

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ABSTRACT

This paper investigates the effect of thermal radiation and heat transfer on the flow of ferromagnetic fluid on a stretching sheet. The appropriate combination of non-magnetic viscous base fluid, magnetic solid and surfactant composes magnetic fluid in the presence of magnetic dipole. The momentum and energy equations with the interaction of ferromagnetic particles formulate governing equations. Similarity transformation is applied on the governing equations to transform partial differential equations to nonlinear ordinary differential equations. A numerical solution is obtained and the effects of magnetic dipole and thermal radiation on dimensionless velocity, temperature, pressure, skin friction and Nusselt number are illustrated graphically.



## 1. Introduction

Ferrofluids are a smart material, synthesized by introducing ferromagnetic particles in a base liquid. Engineering usages of magnetic fluids fascinated scientists for centuries. A few applications in electrical instruments used commercially are hard disks, rotating X-ray tubes, shafts and rods etc. These fluids are also used as heat controlling agent in electric motors and hi-fi speaker [1]. Sensor, densimeters, accelerometer, pressure transducers [2,3] etc. also use magnetic fluid. In medical sciences it can be used to treat cancer and tumor using alternating magnetic fluid [4,5].

Schlichting [6] may be the first to discuss the flow of magnetic fluid within the boundary layer. Rosensweing [7] studied the effects of magnetization. A study of saturated ferrofluids, with the effects of thermal and magnetic field gradients was presented by Neuringer [8]. Tzirtzilakis et al. [9] analyzed forced and free convective boundary layer flow of a magnetic fluid over a flat plate under the action of a localized magnetic field. A part from experimental study, nanofluids with ferro particles have gained a considerable importance in engineering applications recently. The numerical and analytical investigation for such fluids is both tedious and complicated task but the solution of such problems is very important in industrial applications. Feng et al. [10] developed an analytical expression for the effective thermal diffusibility. Tangthieng et al. [11] conducted finite element simulations of heat transfer to a ferrofluid in the presence of an external magnetic field. Kefayati [12] analyzed heat dissipation effects on natural convection flow with linearly temperature distribution on ferrofluid. He observes heat transfer decreases by the increment of the nanoscale ferromagnetic particle. Sheikholeslami and Ganji [13] investigated the influence of an external magnetic field on ferrofluid flow and heat transfer in a semi-annulus. Sheikholeslami et al. [14] extended the previous work and investigated the effects of thermal radiation on heat transfer flow of ferrofluid. More recently, Ellahi et al. [15] analyzed magnetohydrodynamic flow of natural convective nanofluid along a vertical cone with variable wall temperature. Kandelousi and Ellahi [16] use lattice Boltzmann method to simulation ferrofluid flow for magnetic drug targeting. Sheikholeslami et al. [17] investigated flow of nanofluid spraying on an inclined rotating disk for cooling process. Heat transfer effects of nanofluid flow have been studied by several authors [18–23] lately show the value of the subject.

The flow of viscous fluid over stretching surface is one of the active research field because of its extensive applications to many engineering and industrial problems such as, polymer extrusion, wire drawing, manufacturing of food and paper, glass fiber production, and stretching of plastic films. Crane [24] is considered to be the first to work with linearly stretching surface with quiescent surrounding. Further Andreson et al. [25] extended the work of Crane [24] and performed the magnetohydrodynamic (MHD) flow of an electrically conducting power-law fluid over a stretching sheet in the presence of a uniform transverse magnetic field. Vajravelu et al. [26] analyzed the hydromagnetic flow of a second grade fluid over a stretching sheet. Cortell [27] has studied

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the MHD flow of a power-law fluid over a stretching sheet. Abel and Mahesha [28] have demonstrated the flow and heat transfer in a visco-elastic boundary layer flow over a stretching sheet with prescribed surface temperature (PST) case and prescribed heat flux (PHF). A number of studies have been reported over the years regarding stretching surfaces [29–31] indicating the importance of such flow problem.

Literature review reveals that there are only few studies related to heat transfer from ferrofluids. Khan et al. [32] investigate the stagnation point flow and heat transfer of a ferrofluid toward a stretching sheet. They used three types of ferroparticles, magnetite ($Fe_3O_4$), cobalt ferrite ($CoFe_2O_4$) and Mn–Zn ferrite with water and kerosene as conventional base fluids. Qasim et al. [33] have studied the magnetohydrodynamic (MHD) flow of ferrofluid along a stretching cylinder with velocity slip and prescribed surface heat flux boundary. None of these studies incorporates the effects of magnetization and magnetic dipole on the flow of ferro magnetic fluid.

The aim of the present work is to investigate the effects of magnetic dipole and thermal radiation in the boundary layer flow of ferromagnetic fluid past a stretching sheet. The effects of several important parameters like Prandtl number, ferromagnetic interaction parameter, radiation parameter are studied with the help of suitable graphs and table.

## 2. Mathematical formulation

Consider a steady two-dimensional flow of an incompressible, viscous and electrically non-conducting ferromagnetic fluid driven by an impermeable sheet in the horizontal direction shown schematically in Fig. 1. By applying two equal and opposite forces along the horizontal direction which is taken as the x-axis, with the y-axis in a direction normal to the flow, the sheet is stretched with a velocity $u_w(x)$ which is proportional to the distance from the origin. A magnetic dipole is located with its centre on the y-axis at a distance 'a' from the sheet. The magnetic field due to the dipole points in the positive x-direction gives rise to a magnetic field of sufficient strength to saturate the ferrofluid. The sheet temperature $T_w$ is maintained below the Curie temperature $T_c$, while the fluid elements far from the sheet are assumed to be at a temperature $T_\infty = T_c$, and hence incapable of being magnetized until they begin to cool upon entering the thermal boundary layer region adjacent to the sheet.

The boundary layer equations governing the flow and heat transfer in a ferrofluid are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (1)$$

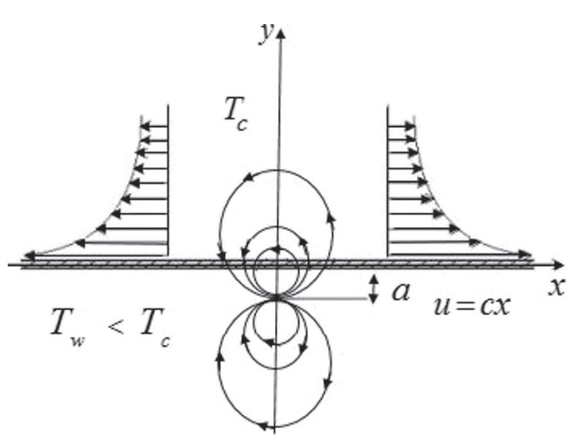


**Fig. 1.** Schematic representation of flow configuration. The circles represent the magnetic field.

**Table 1**
Comparison of $-\theta_1'(0)$ values when $\beta=0, N=0$.

| Pr | Chen [36] | Ishak et al. [37] | Present results |
|---|---|---|---|
| 0:72 | 1.0885 | | 1.08862 |
| 1:0 | 1.3333 | 1.3333 | 1.33333 |
| 3 | 2.5097 | 2.5097 | 2.50972 |
| 10 | 4.7968 | 4.7969 | 4.79682 |

$$p\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_0 M\frac{\partial H}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \quad (2)$$

$$p\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+u_0 M\frac{\partial M}{\partial T}+u\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \quad (3)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)+\mu_0 T\frac{\partial M}{\partial T}\left(u\frac{\partial H}{\partial x}+v\frac{\partial H}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}, \quad (4)$$

where $u$ and $v$ are the velocity components along $x$ and $y$ directions respectively, $\rho$ is the fluid density, $\mu$ is the dynamic viscosity, $\upsilon=\frac{\mu}{\rho}$ is the kinematic viscosity, $c_p$ is the specific heat at constant pressure, $k$ is the thermal conductivity, $q_r$ is the radiative flux, $\mu_0$ is the magnetic permeability, $M$ is the magnetization, $H$ is the magnetic field and $T$ is the temperature of the fluid. The terms $\mu_0 M\frac{\partial H}{\partial x}$ and $\mu_0 M\frac{\partial H}{\partial y}$ in Eqs. (2) and (3), represent the components of the ferromagnetic force per unit volume and depend on the existence of the magnetic gradient. When the magnetic gradient is absent these forces vanish. The second term, on the left-hand side of the thermal energy Eq. (4), accounts for heating due to adiabatic magnetization. The term $\frac{\partial q_r}{\partial y}$ shows the radiation effects.

The radiation flux vector $q_r$ can be found from Isachenko et al.[34] and its formula is derived on the basis of the diffusion concept of radiation heat transfer in the following way:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \quad (5)$$

where $\sigma^*$ is the Stefan–Boltzmann constant and $k^*$ is the mean absorption coefficient. Assuming that the differences in temperature within the flow are such that $T^4$ can be expressed as a linear combination of temperature, expanding $T^4$ in Taylor's series about $T_c$ and neglecting the higher order terms, we get

$$T^4 \cong 4T_c^3 T-3T_c^4. \quad (6)$$

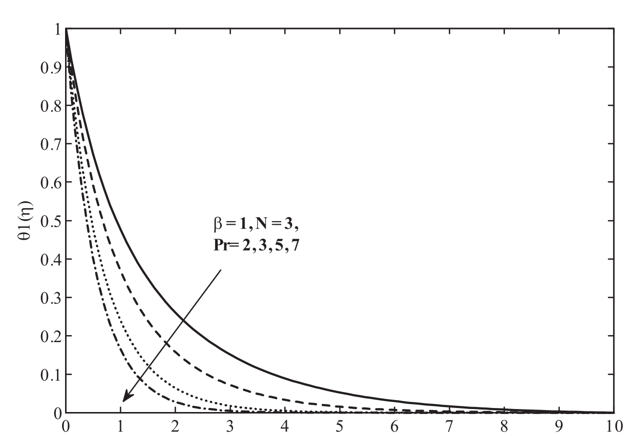


**Fig. 2.** Effect of *Pr* on temperature profile.