Journal of Molecular Liquids 218 (2016) 240–245

Contents lists available at ScienceDirect

Journal of Molecular Liquids

journal homepage: www.elsevier.com/locate/molliq

Slip effects and endoscopy analysis on blood flow of particle-fluid suspension induced by peristaltic wave

M.M. Bhatti [a,*], A. Zeeshan [b], N. Ijaz [b]

[a] Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, China
[b] Department of Mathematics and Statistics, International Islamic University, Islamabad, Pakistan

ARTICLE INFO

Article history:
Received 8 December 2015
Accepted 24 February 2016
Available online xxxx

Keywords:
Slip effects
Particle–fluid suspension
Analytic solution
Blood flow

ABSTRACT

This study describes the endoscopy and slip effects on blood flow of particulate fluid suspension induced by peristaltic wave through a non-uniform annulus. An approximation of long wavelength and low Reynolds is used to model the governing equation of continuity and momentum equation for fluid phase and particulate phase. The influence of all the pertinent parameters has been obtained such as slip parameter (β), Jeffrey fluid parameter (λ_1), particle volume fraction (C) and volume flow rate $Q(z,t)$, velocity constant (v_0) and the inner radius (ϵ) against pressure gradient, pressure rise, friction forces, velocity of fluid phase and particulate phase (u_f, u_p). Numerical results are carried out for pressure rise and friction forces on the inner and outer tube.

1. Introduction

In the past few years, peristaltic motion of various biological fluids has attracted numerous researchers. In industry, various devices such as finger and roller pumps works on the mechanism of peristalsis. In physiology, peristaltic phenomena involved in vasomotion of small blood vessels, movement of Chyme, urine transport from the kidney to the bladder and in many other reproduction systems. Numerous theoretical and experimental investigations show that various biological fluids such as Chyme, spermatic fluid and blood act like a non-Newtonian fluid. However, this analysis can give an appropriate understanding of peristaltic flow in the ureter, but it provides unsatisfactory results when peristaltic flow is found in small blood vessels [1–13]. The influence of peristaltic flow on non-Newtonian fluid with suspended particles is very much substantial in many clinical applications and for various medical diagnoses. Nowadays, it is an important tool for determining various diseases in a human body in which the fluid moves under the mechanism of peristalsis such as small intestines and stomach [14–19]. Misra and Pandey [20] examined the peristaltic transport of particulate fluid suspension in a tube. They found that reversal of flow arises when the pressure gradient is higher than the critical value and this is due to the presence of solid particles in the fluid. Srivastava and Saxena [21] investigated the peristaltic flow of particulate fluid suspension in asymmetric tube by considering the long wavelength and low Reynolds number approximations. They analyzed that pressure drop decreases when the flow rate decreases and its behavior become opposite due to particle concentration.

The study of non-Newtonian Jeffrey fluid model is very helpful in understanding different physical problems because it has the ability to describe the property of complex fluids such as liquid crystals and blood. Furthermore, the analysis on the flow of mixtures is also very helpful to understand various physical problems in different fields of technical importance which includes combustion, atmospheric fallout, aerosol filtration, lunar ash flows, fluidization, powder technology and sedimentation. In hydrodynamics of biological systems, continuum theory of mixtures is applicable because it gives more appropriate understanding of blood rheology, diffusion of proteins, particle deposition in the respiratory tract and swimming of the microorganism. Srivastava et al. [22] examined the peristaltic flow of physiological fluid in a non-uniform geometry. Mekhiemer [23] analyzed the peristaltic flow of couple stress in an annulus with an endoscope. Srinivas et al. [24] described the effects of chemical reaction and thermal diffusion on pulsating flow under the influence of slip boundary conditions. Recently, Abd-Alla [25] studied the peristaltic flow of non-Newtonian Jeffrey in a tube under the influence of radially magnetic field with an endoscope.

In the above mention studies, various attempts have been made to solve blood flow problems using no slip conditions, although a certain amount of slip occurs in a real system. Slip effects can be catheterized as fluid contains the rare field gases or due to elastic characters [26]. In such type of fluids, slip condition originated due to large amount of tangential traction. The slip condition is found in concentrated polymer solution, biological fluids and molten polymer. Furthermore, it can also be observed a clear layer next to the wall in the transport of dilute suspension of particles. Ellahi and Hussain [27] examined the slip effects on

* Corresponding author.
E-mail addresses: muhammad09@shu.edu.cn, mubashirme@yahoo.com (M.M. Bhatti).

http://dx.doi.org/10.1016/j.molliq.2016.02.066
0167-7322/

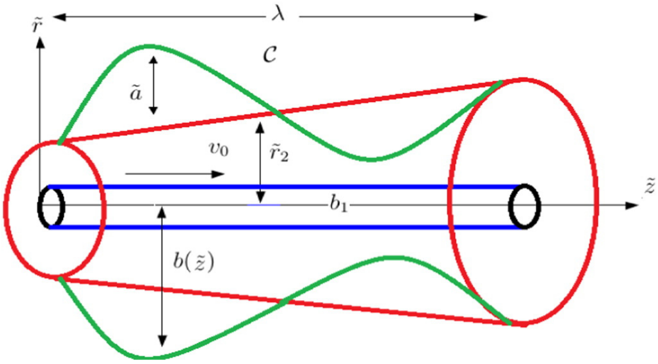

Fig. 1. Geometry of the flow problem.

peristaltic flow of Jeffrey fluid in a rectangular duct. Sinha et al. [28] explored the MHD peristaltic motion with heat transfer in an asymmetric channel under the influence of variable viscosity, slip and temperature jump. A very critical and careful review bear witness that very little effort is devoted to study the particulate-fluid suspension with peristaltic phenomena in an annulus. Some relevant studies on the said topic are available in the list of references [29–41].

With the above analysis in mind, the main objectives of this present investigation are to analyze the effects of slip and endoscopy on peristaltic flow of particulate fluid suspension through a non-uniform annulus. The mathematical model for the present flow problem is applicable for the transport of blood in small vessels. The governing equations of the flow problem are modeled by taking the approximations of creeping flow regime and long wavelength. The resulting differential equations for fluid phase and particulate phase are solved analytically and closed form solution has been obtained. Furthermore, the expressions for pressure rise, friction forces for outer and inner tube are evaluated numerically. This paper is summarized as: after the introduction in Section (1), mathematical formulation of the governing flow problem is described in Sections (2), (3) gives the solution of the problem and finally, Section (4) is devoted to numerical results and discussion.

2. Mathematical formulation

Let us consider two coaxial infinite tubes, the inner gap between both the tube is filled with incompressible, non-Newtonian and irrotational fluid containing a mixture of small spherical particles. The inner tube is taken as rigid whereas the outer tube is considered as non-uniform having peristaltic wave moving with a constant velocity. We have chosen the cylindrical coordinate system $(\tilde{r}, \tilde{z})$ in such a way that $\tilde{r}$ is taken along the radial direction and $\tilde{z}$ is considered along the center of outer and the inner tube as shown in Fig. (1)

The geometry of the wall surface is defined as,

$$\tilde{r}_1 = b_1, \tilde{r}_2 = b(\tilde{z}) + \tilde{a}\sin\frac{2\pi}{\lambda}(\tilde{z} - \mathcal{C}\tilde{t}), \tag{1}$$

where

$$b(\tilde{z}) = b_0 + \mathcal{K}\tilde{z}, \tag{2}$$

In the above equation b_1 is the radius of the inner tube, $b(\tilde{z})$ is the radius of the outer tube at any axial distance $\tilde{z}$ from inlet, b_0 is the radius of the outer tube inlet, $\mathcal{K}(\ll 1)$ is constant whose magnitude depends on the length of the annulus and exit inlet dimensions, $\tilde{a}$ is the wave amplitude, λ is the wavelength, $\mathcal{C}$ is the velocity of the wave propagation and $\tilde{t}$ is the time. The governing equation of motion for fluid phase and particulate phase can be written as.

2.1. Fluid phase

$$(1-C)\frac{\partial \tilde{v}_f}{\partial \tilde{r}} + (1-C)\frac{\partial \tilde{u}_f}{\partial \tilde{z}} + (1-C)\frac{\tilde{v}_f}{\tilde{r}} = 0, \tag{3}$$

$$(1-C)\frac{\partial \tilde{p}}{\partial \tilde{r}} = (1-C)\mu_s\left(\frac{1}{r}\frac{\partial}{\partial r}rS_{rr} + \frac{\partial}{\partial z}S_{rz} - \frac{S_{\theta\theta}}{r}\right) + CS(\tilde{v}_p - \tilde{v}_f), \tag{4}$$

$$(1-C)\frac{\partial \tilde{p}}{\partial \tilde{z}} = (1-C)\mu_s\left(\frac{1}{r}\frac{\partial}{\partial r}rS_{rz} + \frac{\partial}{\partial z}S_{zz}\right) + CS(\tilde{u}_p - \tilde{u}_f), \tag{5}$$

2.2. Particulate phase

$$C\frac{\partial \tilde{v}_p}{\partial \tilde{r}} + C\frac{\partial \tilde{u}_p}{\partial \tilde{z}} + C\frac{\tilde{v}_p}{\tilde{r}} = 0, \tag{6}$$

$$C\frac{\partial \tilde{p}}{\partial \tilde{r}} = CS(\tilde{v}_f - \tilde{v}_p), \tag{7}$$

$$C\frac{\partial \tilde{p}}{\partial \tilde{z}} = CS(\tilde{u}_f - \tilde{u}_p). \tag{8}$$

The stress for Jeffrey fluid is defined as

$$S = \frac{1}{1+\lambda_1}(\dot{\gamma} + \lambda_2\ddot{\gamma}), \tag{9}$$

where

$$S = \frac{9}{2}\frac{\mu_0}{B_0^2}\tilde{\lambda}(C), \mu_s = \frac{\mu_0}{1-\tilde{m}C}, \tilde{\lambda}(C) = \frac{4+\sqrt{8C-3C^2}+3C}{(2-3C)^2}, \tag{10}$$

$$\tilde{m} = 0.70\exp\left[2.49C + \frac{1107}{\tilde{T}}\exp(-1.69C)\right], \tag{11}$$

where B_0 describe the radius of each particle in the fluid, C is the volume fraction density, μ_s is the apparent (or effective) viscosity, S is the drag coefficient, $\tilde{T}$ is the temperature (measured in Kelvin), μ_0 is the fluid viscosity, λ_1 is the ratio between relaxation to retardation time, λ_2 is the delay time, $\dot{\gamma}$ is the shear rate and dot denotes the derivative with

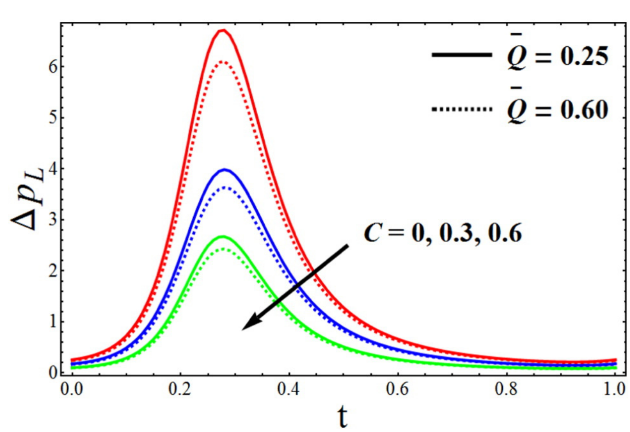

Fig. 2. Pressure rise for different values of C and Q when $\beta = 0.01, \epsilon = 0.32, v_0 = 1, \phi = 0.5, \lambda_1 = 1$.