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Simultaneous effects of nanoparticles and slip on Jeffrey fluid through tapered artery with mild stenosis

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ABSTRACT

This study examines the effects of nanoparticles for the blood flow of Jeffrey fluid in tapered artery with stenosis. The slip effects along with permeable nature of the arterial wall in the presence of convection are also taken into account. Mathematical modeling is based upon continuity, momentum and energy equations. This analysis is carried out under the constraints of mild stenosis. Closed form solutions for velocity and concentration are obtained. Numerical integration is used to analyze the novel features of flow impedance, pressure rise and stream function. Effects of pertinent parameters such as Brownian diffusion coefficient, thermospheric diffusion parameter, Grashof number and material constant of Jeffrey fluid on velocity, temperature and concentration are discussed through graphs.

1. Introduction

Blood is driven through the body by a complex network of veins and arteries. It is constantly in motion as the heart pumps blood through arteries to the different organs and cells of the body. It turns back to heart by veins. The veins are squeezed when muscles in the body contract and push the blood back to the heart. Stenosis is an abnormal narrowing in a blood vessel or other tabular organ or structure. It is also sometimes called a stricture. Most of the times these stenosis cause death when the degree of narrowing becomes significant enough to impede the flow of blood. Due to stenosis in the human artery, the flow of blood is disturbed and resistance to flow becomes higher than that of normal one. The main cause of formation of such stenosis is not yet known clearly but their consequences can be recognized easily.

Complete understanding of flow of blood through arteries of various geometries demands the basics concepts of mechanics of fluid. Some of the basic studies dealing different models of Newtonian and non-Newtonian fluids are given in references [1–6]. Several investigators have highlighted different aspects of blood flow analysis in arteries. Noreen [7] has examined the heat and mass transfer effects on Careau fluid model for blood flow through tapered arteries with stenosis. Mekheimer and El Kot [8] have studied the mathematical modeling of unsteady flow of fluid through anisotropically tapered elastic arteries with time variant overlapping stenosis. They analytically solved their mathematically model for mild stenosis case. Riahi et al. [9] have analyzed the problem of blood flow in an artery in the presence of an overlapping stenosis. A mathematical study on three layered oscillatory blood flow through stenosed arteries has been investigated by Tripathi [10]. Mishra et al. [11] have studied the blood flow through a composite stenosis in an artery with permeable wall.

It is well accepted now that slip effects may appear for two types of fluids (i.e., rare field gases [12] and fluids having much more elastic character). In these fluids, slippage appears as a result of large tangential traction. It is noticed through experiment observations [13–19] that the occurrence of slippage is possible in the non-Newtonian fluids (i.e. polymer solution and molten polymer). In addition, a clear layer is sometimes found next to the wall when flow of dilute suspension of particles is examined. In experimental physiology such a layer is observed when blood flow through capillary vessels is studied [20]. The fluids that exhibit slip effect have many applications, for instance, the polishing of artificial heart valves and internal cavities [21]. Moreover, the slip phenomenon is supported by the molecular theories [22–25].

In recent years, some interest has been promoted to the study of nanofluids. Nanotechnology has been widely used in industry since materials with sizes of nanometers possess unique physical and chemical properties. Nano-scale particle added fluids are called as nanofluid,

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Nomenclature

u,v	velocity components
μ	viscosity
p	pressure
ρ	density
ξ	tapering parameter
δ	height of the stenosis
C_1	concentration
T_1	temperature
R_0	radius of non-tapered artery
d_0	radius of tapered artery
D_B	Brownian diffusion coefficient
D_T	thermospheric diffusion coefficient
N_t	thermophoresis parameter
N_b	Brownian motion parameter
G_r	Grashof number
L	finite length of tube
c	volumetric volume expansion coefficient
l	linear operator
q	embedding parameter

which is firstly introduced by Choi [26]. Choi et al. [27] showed that the addition of a small amount less than 1% by volume of nanoparticles to convectional heat transfer liquids increased the thermal conductivity of the fluid up to approximately two times. Khanafer et al. [28] seem to be the first who have examined heat transfer performance of nanofluids inside an enclosure taken into account the solid particle dispersion. After these studies, nanotechnology is considered by many to be one of the significant forces that drive the next major industrial revolution in the entire world. Currently, convective heat transfer in nanofluids is a topic of major contemporary interest in biological sciences. Some numerical and experimental studies on nanofluids include thermal conductivity are listed in [29–33].

In short, no such analysis is available in the literature which can describe the combined effects of slip, heat convection and nanoparticles for the flow of Jeffrey fluid in the tapered artery with stenosis. Motivated by these facts, the present work has been undertaken for the said purpose. To derive the solutions of nonlinear coupled equations, we have used one of the most effective methods, homotopy perturbation method (HPM). This method is not only valid for small (or large) values of physical parameter but also provides us a simple way to ensure the convergence of series solutions of a nonlinear problem. Some relevant studies on the topic can be seen from the list of references [34–37]. The paper is organized as follows. Section 2 contains the formulation of the problem. In Section 3 solution of the problem is obtained using HPM. Results and discussion are given in Section 4. This section devoted to analyzed, four cases namely, impedance variation, shear stress, variation in velocity distribution and variation in temperature and concentration distributions. The trapping phenomenon is also presented at the end. The conclusion is given in Section 5. Finally the physical features of the major parameters have been illustrated through graphs.

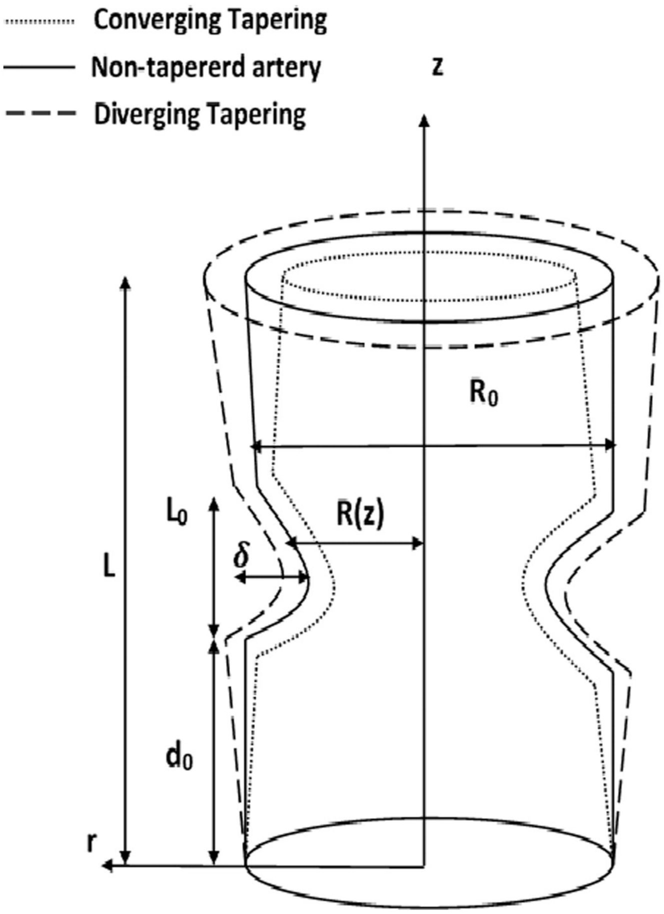

Fig. 1. Geometry of the stenosed tapered artery for different tapering angle.

2. Formulation of the problem

Consider an incompressible nanofluid of viscosity μ and density ρ flowing through a tube having finite length L with overlapping stenosis. Let (r,θ,z) be the coordinates of a material point in the cylindrical polar coordinate system. Here $z-$ axis is taken along the axis of artery while r,θ are along the radial and circumferential direction respectively. Moreover $r=0$ is taken as the axis of symmetry of the tube. Heat and nanoparticle phenomenon are taken into account by giving temperature T_1 and concentration C_1 to the wall of the tube. At the centre of the tube, we consider symmetric conditions for velocity, temperature and concentration. The geometry of the arterial wall of overlapping stenosis for different tapering angles [38,39] is defined as

$$R(z) = d(z)\left[1-\psi\left(L_0^{n-1}(z-d_0)-(z-d_0)^n\right)\right], d_0 < z \leq d_0 + L_0, \qquad (1)$$
$$= d(z), \text{otherwise}$$

with

$$\psi = \frac{\delta n^{\frac{n}{n-1}}}{R_0 L_0^n (n-1)}, \qquad (2)$$

$$d(z) = R_0 + \xi z, \qquad (3)$$

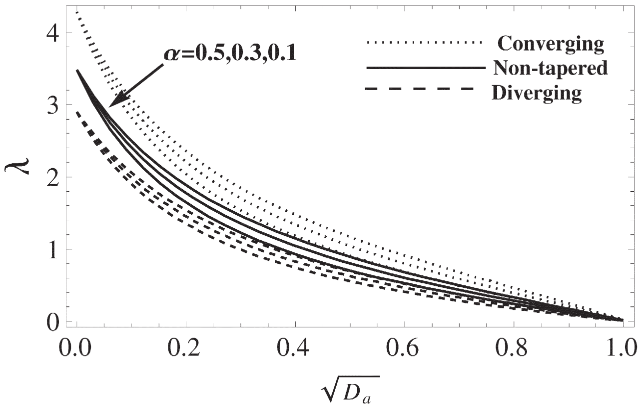

Fig. 2. Variation in impedance λ against $\sqrt{D_a}$ for $\alpha=0.1,0.3,0.5$.