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On squeezing flow of nanofluid in the presence of magnetic field effects

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ABSTRACT

This article investigates the magnetohydrodynamic (MHD) squeezing flow of nanofluid over a porous stretching surface. Constitutive expressions of viscous fluid are employed in the mathematical formulation. Brownian motion and thermophoretic diffusion of nanoparticles are taken into account. Fluid is electrically conducted in the presence of an applied magnetic field. The induced magnetic field is neglected for a small magnetic Reynolds number. Appropriate transformations yield a coupled nonlinear ordinary differential system. The resulting nonlinear system is solved successfully. Graphs are plotted to examine the impacts of physical parameters on the velocity, temperature and nanoparticle concentration distributions. Skin friction coefficient, Nusselt and Sherwood numbers are analyzed numerically.

1. Introduction

Squeezing flow between parallel walls has attracted recent scientists and engineers. Such interest, in fact is due to the occurrence of squeezing flow in the engineering applications like liquid-metal lubrication, food and polymer industries, compression and injection shaping etc. The lubrication system can be also modeled using squeezing flow. The seminal attempt on the topic under lubrication approximation has been reported by Stefan [1]. Leider and Bird [2] investigated the squeezing flow of power-law fluid between the parallel disks. Influence of suction/blowing on the squeezed flow was investigated by Hamza and MacDonald [3]. Heat transfer analysis for unidirectional squeezing flow between parallel disks was performed by Duwairi et al. [4]. Squeezed flow with heat transfer over a porous plate has been investigated by Mahmood et al. [5]. Two-dimensional and axisymmetric squeezing flows between parallel plates have been studied by Rashidi et al. [6]. Siddiqui et al. [7] explored the effects of magnetic field in the squeezing flow between infinite parallel plates due to the normal motion of the plates. Homotopy perturbation method (HPM) has been applied to obtain the analytic solutions of the modeled nonlinear problems. Domairry and Aziz [8] provided the homotopy perturbation solution (HPM) for magnetohydrodynamic (MHD) squeezed flow between the parallel disks. Qayyum et al. [9] discussed the unsteady squeezing flow of Jeffrey fluid between the parallel disks. The squeezing flow of second grade fluid between the parallel disks has been analyzed by Hayat et al. [10]. Sheikholeslami and Ganji [11] explored the squeezing flow of Cu-water nanofluid with the help of homotopy perturbation method. In another article, Sheikholeslami et al. [12] investigated the hydrodynamic squeezing flow of five different nanofluids by the Adomian decomposition method. Domairry and Hatami [13] employed the differential transform method to study the unsteady squeezing flow of Cu-water nanofluid between the parallel plates. Famileh et al. [14] studied the entropy generation characteristics of the squeeze film air damping in a torsional micromirrors.

Nanofluid being a mixture of the nanoparticle and the base fluid is a new variety of energy transport fluid. The nanofluids in view of the extraordinary thermal conductivity enhancement have been useful in several engineering and industrial applications. Cooling rate requirements cannot be achieved by the use of ordinary heat transfer fluids because such fluids have lower thermal conductivity. Thermal conductivity and thermal performance of ordinary heat transfer fluids can be enhanced by submerging the nanoparticles. Novel properties of nanofluids make them potentially applicable in various processes of heat transfer like microelectronics, fuel cells, hybrid-powered engines, etc. It is hoped that the magnetohydrodynamic (MHD) analysis of nanofluids has importance in optical gratings, optical switches, ink float separation, cancer therapy and to guide the particles up in the bloodstream to a tumor with magnets. Masuda et al. [15] studied that the variations in the thermal conductivities and viscosities of liquids through the dispersion of ultra-fine particles in the base fluids. Choi [16] examined that the presence of nanoparticles in the base fluid enhances the thermal properties of fluids. Eastman et al. [17] discussed an abnormal increase in the

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thermal conductivity of ethylene glycol based nanofluids. Buongiorno [18] developed a mathematical model to study the thermal properties of base fluids. This model involves the Brownian motion and thermophoresis effects. After that the researchers investigated the flow of nanofluid under different conditions and different types of nanoparticles. Natural convective boundary layer flow of nanofluid past a vertical flat plate using Buongiorno's model has been investigated by Kuznetsov and Nield [19]. Khan and Pop [20] explored the two-dimensional flow of nanofluid over a linearly stretching sheet. They computed the numerical solutions of the modeled differential system through Keller-box method and provided a detailed analysis of Brownian motion and thermophoresis effects on the heat transfer characteristics. Makinde and Aziz [21] extended this work by assuming convective boundary conditions. They showed that the strength of convective heating has a remarkable impact on the thermal boundary layer. Stagnation-point flow of nanofluid towards a stretching surface has been investigated by Mustafa et al. [22]. Heat and mass transfer effects on the hydromagnetic flow of a viscous fluid with slip conditions and different types of nanoparticles have been considered by Turkyilmazoglu [23]. He constructed both exact and analytical solutions for the resulting flow problems. Hashmi et al. [24] analyzed the analytic solutions for squeezing flow of nanofluid between parallel disks. Second law analysis in the steady flow of nanofluid towards a rotating porous disk is addressed by Rashidi et al. [25]. Zeeshan et al. [26] examined the flow of viscous nanofluid between the concentric cylinders. MHD nanofluid flow and heat transfer in a rotating system have been explored by Sheikholeslami et al. [27]. Bovand et al. [28] studied the flow and heat transfer characteristics of nanofluid over an equilateral triangular obstacle with different orientations. Rashidi et al. [29] examined the magnetohydrodynamic (MHD) two-dimensional flow around a solid square cylinder. The authors solved the governing system numerically through the finite volume method (FVM). Bovand et al. [30] addressed the two-dimensional Darcy–Forchheimer flow around a porous cylinder in the presence of a magnetic field. Rashidi et al. [31] discussed the flow of nanofluid by an equilateral triangular obstacle. The authors performed an optimization analysis to find the optimum conditions for the maximum heat transfer rate and the minimum drag coefficient. Rashidi and Esfahani [32] investigated the forced convection heat transfer in a horizontal channel with a built-in square obstacle. Magnetic field effect is further considered in this investigation.

The purpose of present study is to analyze the magnetohydrodynamic (MHD) squeezing flow of viscous fluid in the presence of nanoparticles. The lower wall of the channel is permeable and stretched. The upper impermeable wall moves towards the lower wall with a specified time-dependent velocity. Mathematical formulation involves the effects of Brownian motion (Dufour effect) and thermophoresis (Soret effect) [33]. The series solutions to the resulting nonlinear differential systems are constructed through the homotopy analysis method (HAM) [34–40]. Computations are performed and analyzed for the physical quantities of interest like skin friction coefficient, Nusselt and Sherwood numbers.

2. Mathematical formulation

Consider the unsteady two-dimensional flow of an incompressible viscous nanofluid between two parallel walls separated by a distance $\sqrt{\nu(1-\gamma t)/a}$. The upper wall at $y=h(t)=\sqrt{\nu(1-\gamma t)/a}$ is moving with velocity $-\frac{\gamma}{2}\sqrt{\frac{\nu}{a(1-\gamma t)}}$ while the lower porous wall at $y=0$ is stretching with velocity $ax/(1-\gamma t)$ $(t<1/\gamma)$. Note that the steady state case of linearly stretching is recovered when $\gamma=0$. The fluid is electrically conducting in the presence of magnetic field $B_0/(1-\gamma t)$ applied in the y– direction. In addition, the electric field and Hall effects are ignored. The induced magnetic field is not considered for a small magnetic Reynolds number. Thermophoresis and Brownian motion effects are retained. All the thermophysical properties are constant. The schematic diagram for the flow analysis is shown in Fig. 1. The governing expressions of mass, momentum, energy and nanoparticle concentration are

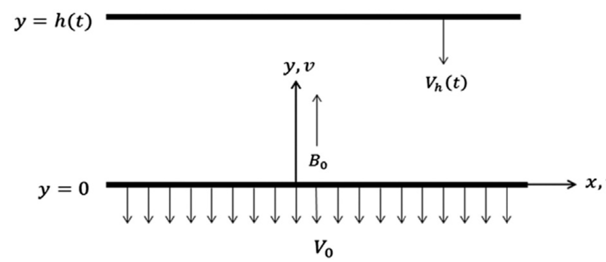

Fig. 1. Geometry of the problem.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p^*}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho_f(1-\gamma t)}u, \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_f}\frac{\partial p^*}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{3}$$

$$\begin{aligned}\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{(\rho c)_p}{(\rho c)_f}D_B\left(\frac{\partial T}{\partial x}\frac{\partial C}{\partial x}+\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right)\\+\frac{(\rho c)_p}{(\rho c)_f}\frac{D_T}{T_m}\left(\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right),\end{aligned} \tag{4}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_m}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{5}$$

where u and v are the velocity components in the x– and y–directions respectively, $\nu(=\mu/\rho_f)$ is the kinematic viscosity, μ is the dynamic viscosity, ρ_f is the density of base fluid, p^* is the pressure, σ is the electrical conductivity, T is the temperature, $\alpha(=k/(\rho c)_f)$ is the thermal diffusivity, $(\rho c)_p$ is the effective heat capacity of nanoparticles, $(\rho c)_f$ is the heat capacity of the fluid, D_B is the Brownian diffusion coefficient, C is the nanoparticle concentration, D_T is the thermophoretic diffusion coefficient and T_m is the mean temperature.

The boundary conditions are

$$\left.\begin{aligned}&u=U_0=\frac{ax}{1-\gamma t}, v=-\frac{V_0}{1-\gamma t}, T=T_0, C=C_0 \text{ at } y=0,\\&u=0, v=V_h=\frac{dh}{dt}=-\frac{\gamma}{2}\sqrt{\frac{\nu}{a(1-\gamma t)}}, T=T_0+\frac{T_0}{1-\gamma t}, C=C_0+\frac{C_0}{1-\gamma t} \text{ at } y=h(t)\end{aligned}\right\} \tag{6}$$

where 'a' denotes the stretching rate of the lower plate, $V_0>0$ indicates the suction and $V_0<0$ for the injection/blowing velocity, and T_0 and C_0 are the temperature and nanoparticle concentration at the lower wall. In boundary conditions (6), $U_0=ax/(1\text{-}\gamma t)$ at $y=0$ indicates that velocity of lower (extensible) plate varies linearly with distance from the origin. Such assumption is realistic in many processes including the extrusion process in which material properties and in particular the elasticity of the extruded sheet is being pulled out by a constant force.

We define the transformations

$$\left.\begin{aligned}&u=U_0F'(\eta), v=-\sqrt{\frac{a\nu}{1-\gamma t}}F(\eta), \eta=\frac{y}{h(t)},\\&T=T_0+\frac{T_0}{1-\gamma t}\theta(\eta), C=C_0+\frac{C_0}{1-\gamma t}\phi(\eta).\end{aligned}\right\} \tag{7}$$