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Impact of magnetic field in three-dimensional flow of an Oldroyd-B nanofluid

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ABSTRACT

This study explores the three-dimensional boundary layer flow of an Oldroyd-B nanofluid. Flow is induced by a bidirectional stretching surface. Brownian motion and thermophoresis effects are considered. Fluid is electrically conducted in the presence of an applied magnetic field. Newly proposed boundary condition requiring zero nanoparticle mass flux at the boundary is employed. The governing nonlinear boundary layer equations through appropriate transformations are reduced into the nonlinear ordinary differential systems. The resulting nonlinear system has been solved for the velocities, temperature and nanoparticle concentration expressions. The contributions of various interesting parameters are studied graphically. The local Nusselt number is tabulated and discussed.

1. Introduction

Nanofluid is the mixture of nanoparticles such as Cu, Ag, TiO_2, or Al_2O_3 and the base liquids like ethylene glycol, oil or water. The nanofluids are expected to have better thermal efficiency than the base liquids. The nanofluids in view of better thermal efficiency have been useful in many industrial and engineering processes including industrial cooling, new types of fuels, vehicle cooling, reduction of fuel in electric power generation plant, imaging and sensing, cancer therapy and many others. Further the magneto nanofluids are quite useful in MHD power generators, petroleum reservoirs, wound treatment, gastric medications and sterilized devices. The variations in thermal conductivities and viscosities of liquids through the dispersion of ultra-fine particles in the base fluids have been analyzed by Masuda et al. [1]. Choi [2] found that the presence of nanoparticles in the base fluid enhances the thermal properties of fluids. Buongiorno [3] developed a mathematical model to explore the thermal properties of base fluids. He used Brownian motion and thermophoresis effects to enhance the thermal properties of base liquids. Stagnation-point flow of nanofluid towards a stretching sheet is examined by Mustafa et al. [4]. Turkyilmazoglu [5] analyzed the heat and mass transfer effects in hydromagnetic flow of viscous fluid. Here slip conditions and various types of nanoparticles are considered. The unsteady natural convection flow of some nanofluids past a vertical infinite flat plate with radiation effect is analyzed by Turkyilmazoglu and Pop [6]. Makinde et al. [7] examined the buoyancy driven flow of magneto nanofluid near a stagnation point. Here flow is caused due to convectively heated stretching/shrinking sheet. Entropy generation analysis in the steady MHD flow of nanofluid by a rotating porous disk is examined by Rashidi et al. [8]. Sheikholeslami and Ganji [9]explored the heat transfer effect in Cu–water nanofluid flow between the parallel plates. Hayat et al. [10] numerically studied the peristaltic flow of viscous nanofluid. The MHD free convection flow of nanofluid in an eccentric semi-annulus is discussed by Sheikholeslami et al. [11]. Recently Kuznetsov and Nield [12] discussed the natural convection flow of nanofluid past a vertical plate. Marangoni convection flow of pseudoplastic nanofluids with heat transfer is examined by Lin et al. [13]. They considered the variable thermal conductivity and radiation effects. Zhang et al. [14] explored the MHD flow and heat transfer of nanofluids in porous media with variable surface heat flux, thermal radiation and chemical reaction. Sheikholeslami and Ganji [15] examined the MHD flow and heat transfer of nanofluid between the parallel plates. Sheikholeslami et al. [16] explored the magnetic field effects on the flow of nanofluid in a cubic cavity. Two-phase simulation of nanofluid between the two coaxial cylinders in the presence of magnetic field effects is investigated by Sheikholeslami and Abelman [17]. Lin et al. [18] studied the MHD flow

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and heat transfer of pseudo-plastic nanofluid past a stretching surface with internal heat generation. Shehzad et al. [19] addressed the mixed convective peristaltic transport of nanofluids in the presence of viscous dissipation and heat generation/absorption. Three-dimensional rotating flow of nanofluid over a linearly stretching disk is reported by Mustafa et al. [20].

The boundary layer flow induced by the stretching of a sheet has many applications in the industries including manufacturing of plastic and rubber sheets, annealing and thinning of copper wires, drawing on stretching sheets through quiescent fluids, continuous cooling of fiber spinning, polymer filament or sheet extruded from a dye, boundary layer along a liquid film condensation process, aerodynamic extrusion of plastic films and many others [21–23]. Examples of non-Newtonian fluids include paints, mud, paper pulp, shampoos, applesauce, ketchup, slurries, certain oils and polymer solutions. A single constitutive relationship is not sufficient to explore the diverse characteristics of non-Newtonian materials. Different models of non-Newtonian fluids have been developed in the past. Generally, non-Newtonian fluids are divided into three categories, namely the differential, rate and integral types. The fluid model under consideration is a subclass of rate type fluids. An Oldroyd-B fluid model characterizes both the relaxation and retardation time's effects. Tan and Masuoka [24] analyzed the Stokes first problem for an Oldroyd-B fluid in a porous half space. Unsteady helical flows of generalized Oldroyd-B fluid are examined by Tong et al. [25]. Energetic balance to the flow of an Oldroyd-B fluid has been investigated by Fetecau et al. [26]. Here the plate is subjected to a time-dependent shear stress. The boundary layer flow of an Oldroyd-B fluid in the region of stagnation point has been examined by Sajid et al. [27]. Jamil et al. [28] explored the unsteady helical flows of Oldroyd-B fluid. Zheng et al. [29] carried out a study to analyze the slip effect on MHD flow of generalized Oldroyd-B fluid. The MHD flow of an Oldroyd-B fluid through a porous channel is reported by Hayat et al. [30]. Thermophoresis particle deposition in mixed convection three-dimensional radiative flow of an Oldroyd-B fluid has been investigated by Shehzad et al. [31].

The aim of this article is to address the magneto hydrodynamic (MHD) three-dimensional flow of an Oldroyd-B fluid with nanoparticles. Effects of Brownian motion and thermophoresis at the surface are considered. To our knowledge, no such analysis of Oldroyd-B fluid is reported yet in the literature. This article is written in the following fashion. The next section discusses the mathematical formulation under boundary layer and low magnetic Reynolds number assumptions. Section 3 consists of computations for solutions. Sections 4 and 5 examine the convergence analysis and discussion of results. Main observations of this work are listed in Section 6.

2. Mathematical formulation

We consider the steady three-dimensional flow of an incompressible Oldroyd-B nanofluid. The flow is caused by a bidirectional stretching surface. The fluid is considered electrically conducting in the presence of constant magnetic field B_o applied in the z-direction. Hall and electric field effects are absent. The induced magnetic field is not considered for a small magnetic Reynolds number. Brownian motion and thermophoresis effects are taken into account. We adopt the Cartesian coordinate system in such a way that x- and y-axes are taken in the direction of motion and z-axis is normal to it (see Fig. 1). The sheet at $z = 0$ is stretched in the x- and y-directions with velocities U_w and V_w respectively. The thermophysical properties of fluid are taken constant. The boundary layer expressions governing the flow of an Oldroyd-B nanofluid are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

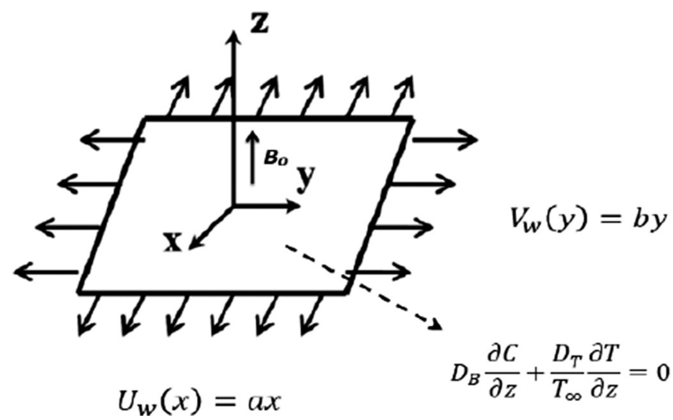

Fig. 1. Flow configuration.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \lambda_1\left(\begin{array}{c} u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + w^2\frac{\partial^2 u}{\partial z^2} \\ +2uv\frac{\partial^2 u}{\partial x\partial y} + 2vw\frac{\partial^2 u}{\partial y\partial z} + 2uw\frac{\partial^2 u}{\partial x\partial z} \end{array}\right)$$
$$= \nu\left(\frac{\partial^2 u}{\partial z^2} + \lambda_2\left(\begin{array}{c} u\frac{\partial^3 u}{\partial x\partial z^2} + v\frac{\partial^3 u}{\partial y\partial z^2} + w\frac{\partial^3 u}{\partial z^3} \\ -\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial z^2} - \frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2} \end{array}\right)\right) - \frac{\sigma B_0^2}{\rho_f}\left(u + \lambda_1 w\frac{\partial u}{\partial z}\right), \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \lambda_1\left(\begin{array}{c} u^2\frac{\partial^2 v}{\partial x^2} + v^2\frac{\partial^2 v}{\partial y^2} + w^2\frac{\partial^2 v}{\partial z^2} \\ +2uv\frac{\partial^2 v}{\partial x\partial y} + 2vw\frac{\partial^2 v}{\partial y\partial z} + 2uw\frac{\partial^2 v}{\partial x\partial z} \end{array}\right)$$
$$= \nu\left(\frac{\partial^2 v}{\partial z^2} + \lambda_2\left(\begin{array}{c} u\frac{\partial^3 v}{\partial x\partial z^2} + v\frac{\partial^3 v}{\partial y\partial z^2} + w\frac{\partial^3 v}{\partial z^3} \\ -\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial z^2} - \frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial z^2} - \frac{\partial v}{\partial z}\frac{\partial^2 w}{\partial z^2} \end{array}\right)\right) - \frac{\sigma B_0^2}{\rho_f}\left(v + \lambda_1 w\frac{\partial v}{\partial z}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial z^2} + \frac{(\rho c)_p}{(\rho c)_f}\left(D_B\left(\frac{\partial T}{\partial z}\frac{\partial C}{\partial z}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2\right), \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D_B\left(\frac{\partial^2 C}{\partial z^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial z^2}\right). \tag{5}$$

The boundary conditions for the present flow analysis are [12,31]:

$$u = U_w(x) = ax, v = V_w(y) = by, w = 0, T = T_w(x), D_B\frac{\partial C}{\partial z} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial z} = 0 \text{ at } z = 0, \tag{6}$$

$$u \to 0, \quad v \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as } z \to \infty, \tag{7}$$

where u, v and w are the velocity components in the x-, y- and z-directions respectively, $\nu(=\mu/\rho_f)$ the kinematic viscosity, μ the dynamic viscosity, ρ_f the density of base fluid, λ_1 the relaxation time, λ_2 the retardation time, σ the electrical conductivity, T the temperature, $\alpha = k/(\rho c)_f$ the thermal diffusivity of fluid, k the thermal conductivity, $(\rho c)_f$ the heat capacity of fluid, $(\rho c)_p$ the effective heat capacity of nanoparticles, D_B the Brownian diffusion coefficient, C the nanoparticles concentration, D_T the thermophoretic diffusion coefficient, T_w and T_∞ the temperatures of the surface and far away from the surface and C_∞ the nanoparticle concentration far away from the surface. The subscript w denotes the wall