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Analytical solution of magnetohydrodynamic flow of Jeffrey fluid through a circular microchannel

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ABSTRACT

The magnetohydrodynamic (MHD) flow of Jeffrey fluid in a circular microchannel is presented. Using the method of variable separation, the analytical solutions to both DC-operated MHD and AC-operated MHD micropumps are found. The flow is assumed to be laminar, unidirectional, one dimensional and driven by the Lorentz force. The Lorentz force can be taken as hydrostatic pressure gradient in the momentum equation of the MHD microchannel flow model. The effects of Hartmann number *Ha*, dimensionless relaxation time $\overline{\lambda}_1$ and retardation time $\overline{\lambda}_2$ on the velocity and volumetric flow rate are investigated. The velocity and volumetric flow rate grow and then reduce with Hartmann number *Ha*. There is a critical value of the *Ha* for MHD velocity and an optimum *Ha* for maximum volumetric flow rate. In addition, a comparison with previous works is also provided to confirm the validity of the present results.

1. Introduction

A lot of research efforts have been performed in microfluidic systems [1–7], especially the MHD micropump. It is vital because of having no moving parts, its simple fabrication processes, bidirectional pumping ability, lower actuation voltages, reduced risk of mechanical damage and a continuous fluid flow. The Lorentz force is produced when a conducting fluid flows through the device equipped with the electric field and the transverse magnetic field, which is the pumping source in the MHD micropump system. Due to the prospective application in microfluidic systems [8–12], the MHD micropump has been the main subject of many studies in recent years.

Currently, there are two different versions of MHD micropump available in the market: DC-operated and AC-operated MHD micropumps. Theoretical and experimental studies about DC and AC MHD devices had begun in many years ago. Jang and Lee [13] fabricated a MHD micropump employing a permanent magnet and applying DC current. Furthermore, Ho [14] explored DC MHD micropump theoretically and experimentally and predicted the pumping performance. In order to realize it, an analytical model was provided to analyze the characteristic of MHD flow in a rectangular duct, and the model was based on the steady state, incompressible and fully developed flow theory. Kim et al. [15] investigated the capability of a DC MHD micropump fabricated on photosensitive glass for circulating liquid metal. Homsy et al. [16] described the operation of a DC MHD micropump with high current densities. Lemoff and Lee [17–19] presented theory, fabrication method and experimental results of a AC MHD pump where the Lorentz force was used to propel an electrolytic solution along a microchannel etched in silicon. Eijkel et al. [20] developed an AC MHD micropump for chromatographic application. Zhong et al. [21] used the AC MHD micropump to actuate flow in conduits fabricated with ceramic tapes.

Moghaddam [22] obtained analytical solutions for both DC and AC MHD micropumps with circular cross-section. Very recently, Buren et al. [23] studied the electromagnetohydrodynamic (EMHD) flow parallel to the corrugation grooves. And then, the further research associated with EMHD flow perpendicular to the corrugation grooves was carried out by Buren and Jian [24]. The fluids mentioned above are Newtonian fluids.

Nowadays, most of the fluids used in industry and biomedical treatment are non-Newtonian fluids. More and more researchers presented the MHD flow phenomena of non-Newtonian fluids. Sarpkaya [25] considered the MHD flow of non-Newtonian fluids firstly. Ellahi and Nadeem [26] analyzed the steady flow of non-Newtonian fluids with magnetic field and nonlinear slip effects numerically. Furthermore, Nadeem et al. [27] studied the effects of magnetic field and partial slip on obliquely flow. Moghaddam [28] studied MHD micropump of power-law fluid and considered the effect of power law behavior exponent on volumetric flow rate in a rectangular channel. Moreover, the application of MHD flow has attracted the interest of many scientists to analyze the flow of non-Newtonian fluids through different walls of channels. Hatami et al. [29] investigated MHD flow of nanofluid in

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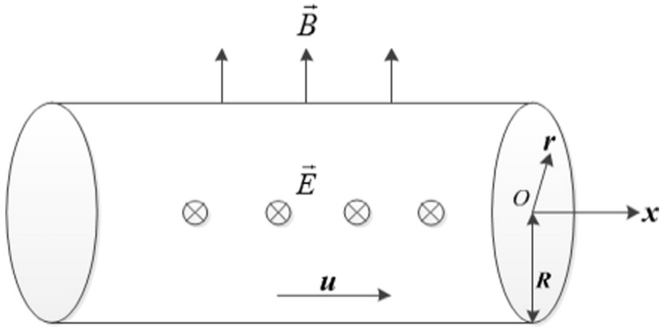

Fig. 1. The schematic of the MHD flow of Jeffrey fluid through a circular microchannel.

non-parallel walls using different analytical methods. Nadeem et al. [30] presented MHD flow of micropolar nanofluid between rotating horizontal parallel plates. Hatami and Ganji [31] analytically and numerically surveyed the MHD flow of nanofluid in the porous medium between two coaxial cylinders. Recently, Nadeem et al. [32] numerically investigated the MHD oblique flow of Walter's B type nanofluid over a convective surface. Zhao et al. [33] focused on the MHD flow of the generalized Maxwell fluid under AC electric field through a two-dimensional rectangular channel. Si and Jian [34] studied the MHD flow of Jeffrey fluid between two slit microparallel plates with corrugated walls by utilizing perturbation technique.

In many literatures, the generalized Jeffrey fluid model was defined as working fluid. Liu et al. [35] presented the AC electroosmotic flow of Jeffrey fluid between two microparallel plates by using the method of separation of variables. Mehmood et al. [36] explored the steady stagnation point flow of Jeffrey fluid toward a stretching surface. As a matter of fact, Jeffrey fluid model treats the classical viscous Newtonian fluid as a special case for $\lambda_1 = 0$, $\lambda_2 = 0$, and as the Maxwell fluid when $\lambda_1 \neq 0$, $\lambda_2 = 0$. Furthermore, the circular channel is very common in practical applications. So in this paper we study the MHD flow of Jeffrey fluid through a circular microchannel and discuss the variations of velocity and volumetric flow rate.

2. Problem statement and mathematical formulation

We consider the MHD flow of an incompressible Jeffrey fluid through a circular microchannel. Geometry of the problem is shown in Fig. 1. The channel has a circular cross section with a radius R and a length L. The channel is subjected to an electrical field E imposed from outside to inside and an uniform magnetic field vertically upward with a strength B. The magnetic and electric fields are perpendicular to each other and produce the so called Lorentz force in axial direction. A cylindrical coordinate system is introduced, where r-axis and x-axis are radical and flow directions, respectively.

The numerical results of MHD flow in 3D microchannels showed that the velocity field is unidirectional in most parts of the microchannels [37]. So the velocity components in the r and θ directions are neglected compared with velocity along the conduit's axis. The continuity equation reduces to $\partial u / \partial x = 0$. The Cauchy momentum equation in x direction can be expressed as [22]:

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} - \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rx}), \tag{1}$$

where u is the axial velocity along positive x direction, ρ is the fluid density, t is the time, p is the pressure and τ_{rx} is the component of stress tensor. For generalized Jeffrey fluid, the constitutive equation satisfies [35]:

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\tau_{rx} = -\eta_0\left(1+\lambda_2\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial r}, \tag{2}$$

where λ_1 is relaxation time, λ_2 is retardation time and η_0 is the zero shear rate viscosity. Combining Eqs. (1) and (2) leads to [34]:

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\rho\frac{\partial u}{\partial t}+\frac{\partial p}{\partial x}\right) = \eta_0\left(1+\lambda_2\frac{\partial}{\partial t}\right)\left(\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}\right). \tag{3}$$

Assuming that hydrostatic pressure head in this study just provides the inlet flow velocity instead of giving the pressure difference along the channel, the pressure difference primarily caused by the Lorentz force $\vec{J}\times\vec{B}$ ($\vec{J}$ is electric current density) and its effect is taken as uniformly distributed over the entire length of the channel L. Hence, the pressure head is expressed as follows [14]:

$$-\frac{\partial p}{\partial x} = \frac{\Delta p}{L}, \tag{4a}$$

where Δp is the pressure head along the channel with length L given by the cross products of current density vector and magnetic field intensity vector. For DC-operated micropump, it can be expressed as:

$$\Delta p = \left(\vec{J}\times\vec{B}\right)L, \tag{4b}$$

where $\vec{B}$ and $\vec{J}$ are related to each other by the Ohm's low:

$$\vec{J} = \sigma\left(\vec{E}+\vec{u}\times\vec{B}\right), \tag{4c}$$

where σ is the electrical conductivity.

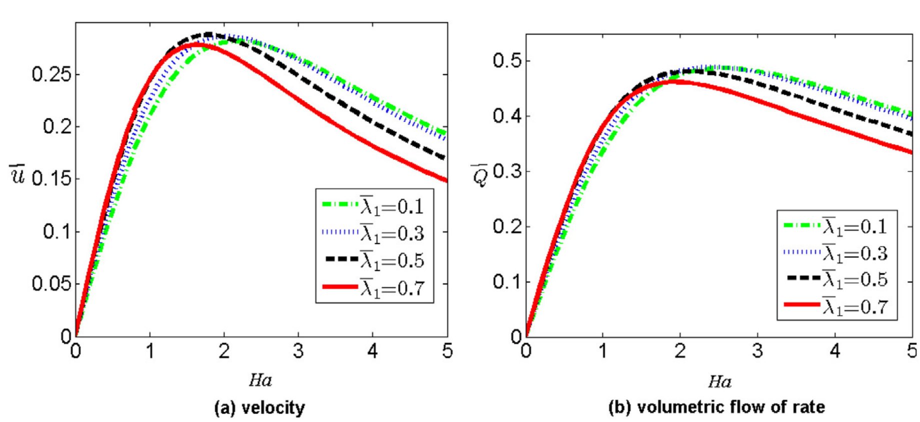

Fig. 2. Variations of DC-operated MHD velocity $\bar{u}$ and volumetric flow of rate $\overline{Q}$ with Ha for different relaxation time $\overline{\lambda}_1$ when $\overline{\lambda}_2 = 0.08$, $\beta = 1$ and $\bar{t} = 1$.