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Peristalsis of silver-water nanofluid in the presence of Hall and Ohmic heating effects: Applications in drug delivery

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ABSTRACT

This study examines the peristaltic transport of silver-water nanofluid in the presence of constant applied magnetic field. Ohmic heating and Hall effects are also taken into account. Present attempt includes the velocity and thermal slip effects. Mathematical modeling is carried out employing the lubrication approach. The resulting system of equations is numerically solved. Effects of sundry parameters on the quantities of interest are studied through graphs. Results show that addition of 5% silver nanoparticles reduces the velocity of base fluid (water in this case) by almost 10% and its temperature by 16%. Further presence of Hall effects lessens the changes brought by an applied magnetic field in the state of nanofluid. Maximum velocity of nanofluid decreases with an increase in the value of velocity slip parameter whereas maximum temperature enhances by increasing the thermal slip parameter. It is hoped that the presented analysis is of considerable importance in the modern drug delivery systems in biomedical engineering.

1. Introduction

Naturally poor thermal conductivity of the conventional fluids e.g. water, oil, ethylene glycol mixture has proven to be principal restriction in enhancing the heat transfer processes involving such fluids. Metals (e.g. silver, copper, gold) usually have higher thermal conductivity when compared to fluids. The idea of utilizing the high thermal conductivity of metals to enhance the thermal conductivity of conventional fluids led to the idea of nanofluids. Such fluids are prepared by suspending the nanometer sized metal particles in the base fluid. As a result of these suspensions the effective thermal conductivity of the solution thus obtained is higher than the thermal conductivity of base fluid. The term nanofluid was initially introduced by Choi [1]. The most commonly used nanoparticles are composed of metal oxides (CuO, $Al2O_3$), metals (Cu, Ag, and Au) and nitride/carbide ceramics (AlN, SiN, SiC, TiC). Base fluids are usually the orthodox fluids e.g. water, ethylene glycol, and oil. Nanofluids have proven to be very useful in numerous industrial and engineering devices. Nanofluids are widely used in numerous industrial and engineering appliances e.g. as coolants in nuclear reactors, automobile and IT industry, in industrial cooling, in extraction of geothermal power, in modern drug delivery systems, in heat exchangers, in diagnostic tests and in cancer therapy (hyperthermia and cryosurgery) and in microscale fluidic applications [2]. Subject to such wide usefulness several experimental and theoretical investigations were carried out to analyze different aspects of nanofluids (see Refs. [3–11]).

Flow within a channel/tube induced due to propagation of sinusoidal waves along the channel walls is termed as peristaltic flow. Such flows find numerous applications in physiology e.g. in the food transport through esophagus, in urine transport from the kidneys to bladder, in bile transport through bile duct, in the perivascular space of the brain, and in the transport of male and female reproductive cells through their reproductive tracks. Due to its utility in preventing the fluid thus transported from contamination, the mechanism of peristalsis is utilized in designing several industrial appliances e.g. roller and finger pumps, hose pumps, pumps in dialysis and heart lung machines and in transport of several corrosive and sensitive fluids in the nuclear industry. Subject to such wide occurring and utility, several investigations examined the peristaltic transport of viscous/non-Newtonian fluids under different flow configurations. Some of these investigations can be seen through Refs. [12–28] and several references therein. Over the past decade, colloidal drug delivery systems have been developed and evolved in order to improve the efficiency and the specificity of drug action. The novel properties of nanoparticles such as small size, customized surface, improved solubility and multi-functionality of nanoparticles open many doors and create new biomedical applications. At high temperatures or pressures the organic antibacterial materials are often less stable. However the inorganic materials such as metal and metal oxides are very helpful under such conditions. This fact further highlights the importance of the nanofluids. Peristaltic transport of nanofluids is of significant importance in modern drug delivery systems in which magnetic

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Table 1
Numerical values of thermo-physical properties of water and silver.

Phase	ρ (kg/m^3)	K (W/mK)	C (J/kg K)	β (l/k) $\times 10^{-6}$	σ (S/m)
Water (H_2O)	997.1	0.613	4179	210	0.05
Silver (Ag)	10,500	429	235	18.9	6.3×10^7

fluxes are used to guide the nanoparticles up against the blood stream to the tumor site. Recent advancements in hyperthermia and cryosurgery as means to destroy the undesired tissues have further enhanced the importance of nanofluids. Under such motivations the peristaltic transport of nanofluids is already studied through some attempts. For instance Hayat et al. [29] studied the slip effects on the peristaltic transport of nanofluid using Buongiorno's [3] model. Same approach is used to study the peristaltic transport of nanofluids in the studies carried out by Abbasi et al. [30] and Shehzad et al. [31]. A review of the available literature suggests that not much has been said about the peristaltic transport of nanofluids using the phase flow model. Some studies examining the peristalsis of nanofluid through this model can be seen by Refs. [32–35]. Hall effects are further significant in the analysis of MHD peristaltic flows under the influence of strong applied magnetic fields. The Hall parameter in such analysis is the ratio of electron-cyclotron frequency and the electron–atom collision frequency. This effect results considerably when the applied magnetic field is strong or when the collision frequency is low. In dealing with weak and moderate magnetic fields, the Hall effect is ignored and the results give good agreement with experimental data. However, when dealing with strong magnetic fields the Hall current is important as it has a considerable effect on the current density and consequently on the Lorentz force term. These facts make the impact of Hall current on the flow worth studying.

With these motivations, the present investigation analyzes the peristaltic transport of silver-water nanofluid using the two-phase flow model under the influence of constant applied magnetic field. The Hall and Ohmic heating effects are also taken into account. Velocity and thermal slip effects are employed in the formulation. Mathematical modeling is carried out using the long wavelength and low Reynolds number approximations. Numerical solutions for the axial velocity, pressure rise per wavelength, pressure gradient, temperature and heat transfer rates at the wall are obtained and analyzed graphically.

2. Problem formulation

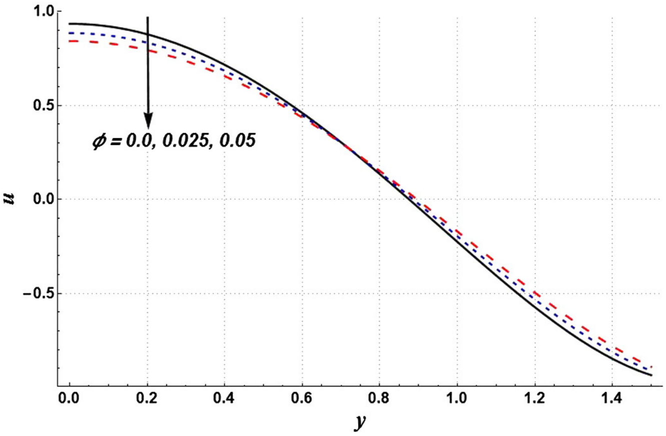

Fig. 1. Effect of nanoparticle volume fraction on the velocity when $Gr = 3.0$, $a = 0.5$, $x = 0$, $\eta = 1.2$, $M = 2.0$, $m = 2.0$, $Br = 0.2$, $\beta_1 = 0.1$, $\gamma = 0.1$ and $\varepsilon = 2.0$.

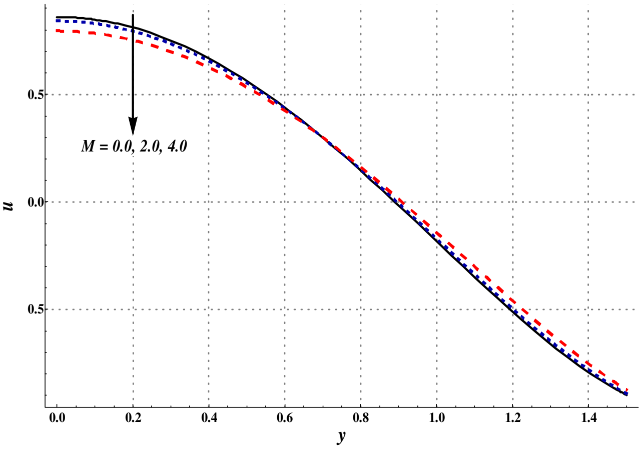

Fig. 2. Effect of Hartman number on the velocity when $Gr = 3.0$, $a = 0.5$, $x = 0$, $\eta = 1.2$, $\phi = 0.05$, $m = 2.0$, $Br = 0.2$, $\beta_1 = 0.1$, $\gamma = 0.1$ and $\varepsilon = 2.0$.

Consider an incompressible electrically conducting nanofluid composed of silver nanoparticles and water in a symmetric channel of width 2d. Water and silver nanoparticles are in thermal equilibrium. Cartesian coordinate system is defined in such a manner that the $\overline{Y}-$ axis is taken normal to the $\overline{X}-$ axis and the $\overline{X}-$ axis lies along the length of the channel. A uniform magnetic field of strength B_0 is applied. Electric field effects are zero. Induced magnetic field is ignored subject to the assumption of low magnetic Reynolds number. The channel walls are maintained at constant temperature T_0. Flow within the channel is generated due to the sinusoidal waves propagating along the channel walls. Geometry of the peristaltic walls is

$$\pm\overline{H}(\overline{X},\overline{t}) = \pm d \pm a_1 \cos\left(\frac{2\pi}{\lambda}(\overline{X}-c\overline{t})\right), \tag{1}$$

where $\pm$ denoted the walls in positive/negative $\overline{Y}-$ directions, a_1 is the amplitude of the peristaltic wave, c is the wave speed, t is the time and λ is the wavelength of peristaltic wave. The Lorentz force is defined by

$$\mathbf{F} = \mathbf{J} \times \mathbf{B}, \tag{2}$$

where $\mathbf{B} = [0, 0, B_0]$ is the applied magnetic field and $\mathbf{J}$ is the current density. For zero applied and induced electric fields the current density taking into account the Hall effects is defined by

$$\mathbf{J} = \sigma_{eff}\left[\mathbf{V} \times \mathbf{B} - \frac{1}{en_e}[\mathbf{J} \times \mathbf{B}]\right]. \tag{3}$$

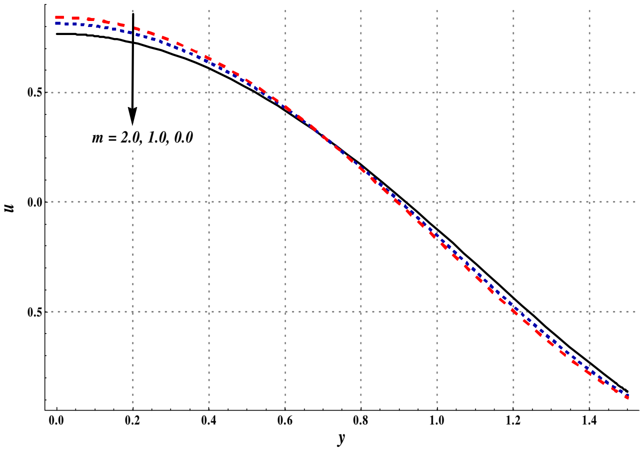

Fig. 3. Effect of Hall parameter on the velocity when $Gr = 3.0$, $a = 0.5$, $x = 0$, $\eta = 1.2$, $\phi = 0.05$, $M = 2.0$, $Br = 0.2$, $\beta_1 = 0.1$, $\gamma = 0.1$ and $\varepsilon = 2.0$.