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Analytical study of micropolar fluid flow and heat transfer in a channel with permeable walls

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ABSTRACT

In this study, micropolar fluid flow in a channel subject to a chemical reaction is investigated analytically using least square method (LSM) and numerically. Some efforts are done to show reliability and performance of the present method compared with the numerical method (Runge–Kutta fourth-order) to solve this problem. The results reveal that the LSM can achieve suitable results in predicting the solution of these problems. Also, the effect of consequential parameters such as Reynolds number (*Re*), micro rotation and Peclet number on the stream function, velocity and temperature distribution is discussed. The results show that the stream function decreases with increase of Reynolds number and velocity boundary layer thickness decreases as *Re* increases. With increase of Peclet number (*Pe*) the oscillation of temperature fluid and concentration profile increases. Furthermore, the effect of the Reynolds and Peclet numbers on Nusselt and Sherwood numbers is completely investigated from the physical view point in the present study.

1. Introduction

The theory of a micropolar fluid derives from the need to model the flow of fluids that contain rotating micro-constituents. A micropolar fluid is the fluid with internal structures which coupling between the spin of each particle and the macroscopic velocity field is taken into account. It is a hydro dynamical framework suitable for granular systems which consist of particles with macroscopic size. Eringen [1] was the first pioneer of formulating the theory of micropolar fluids. His theory introduced new material parameters, an additional independent vector field – the microrotation – and new constitutive equations which must be solved simultaneously with the usual equations for Newtonian fluid flow. Although the field of micropolar fluids is rich in literature, some gaps can be observed and needs more study in this field. For instance, Gorla [2], Rees and Bassom [3] investigated the flow of a micropolar fluid over a flat plate. Also, Kelson and Desseaux [4] studied the flow of micropolar fluids on stretching surfaces. Heat and mass transfer have important role in many industrial and technological processes such as manufacturing and metallurgical processes which heat and mass transfer occur simultaneously. The influence of a chemical reaction and thermal radiation on the heat and mass transfer in MHD micropolar flow over a vertical moving plate in a porous medium with heat generation was studied by Mohamed and Abo-Dahab [5]. Recently effect of using micropolar fluid, nanofluid, etc. on flow and heat transfer has been studied by several authors [6–15].

There are some simple and accurate approximation techniques for solving nonlinear differential equations called the Weighted Residuals Methods (WRMs). Collocation, Galerkin and Least Square Method (LSM) are examples of the WRMs which are introduced by Ozisik [16] for using in heat transfer problem. Stern and Rasmussen [17] used collocation method for solving a third order linear differential equation. Vaferi et al. [18] have studied the feasibility of applying the Orthogonal Collocation method to solve diffusivity equation in the radial transient flow system. Recently Hatami et al. [19] used LSM for heat transfer study through porous fins, also the problem of laminar nanofluid flow in a semi-porous channel in the presence of transverse magnetic field is investigated. Hatami and Ganji [20] found that LSM is more appropriate than other analytical methods for solving the nonlinear heat transfer equations. This method has been successfully applied to solve many types of nonlinear problems [21–24].

In this study LSM is applied to find the approximate solutions of nonlinear differential equations governing the micropolar fluid flow in a channel. A comparison between the LSM results and the numerical solution has been provided. The velocity, temperature and concentration profiles are shown and the influence of Reynolds numbers, micro

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Nomenclature

C	species concentration
D^*	thermal conductivity and molecular diffusivity
f	dimensionless stream function
g	dimensionless micro rotation
h	width of channel
LSM	Least square method
j	micro-inertia density
N	micro rotation/angular velocity
$N_{1,2,3}$	dimensionless parameter
Nu	Nusselt number
Sh	Sherwood number
Sc	Schmidt number
P	pressure
Pr	Prandtl number
Pe	Peclet number
q	mass transfer parameter
Re	Reynolds number
T	fluid temperature
S	micro rotation boundary condition
u, v	Cartesian velocity components
x, y	Cartesian coordinate components parallel & normal to channel axis, respectively

Greek symbols

η	similarity variable
θ	dimensionless temperature
μ	dynamic viscosity
κ	coupling coefficient
ρ	fluid density
ν_s	micro rotation/spin-gradient viscosity
ψ	stream function

rotation/angular velocity and Peclet number on the flow, heat transfer and concentration characteristics is deeply discussed.

2. Problem statement and mathematical formulation

In the present study, the steady laminar flow of a micropolar fluid along a two-dimensional channel with porous walls through is considered which fluid is uniformly injected or removed with speed V_0. The lower channel wall has a solute concentration C_1 and temperature T_1 while the upper wall has solute concentration C_2 and temperature T_2 as shown in Fig. 1. Using Cartesian coordinates, the channel walls are parallel to the x-axis and located at $y = \pm h$ where $2h$ is the channel width. The relevant equations governing the flow are [25]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+(\mu+\kappa)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\kappa\frac{\partial N}{\partial y}, \tag{2}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial y}+(\mu+\kappa)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\kappa\frac{\partial N}{\partial x}, \tag{3}$$

$$\rho\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=-\frac{\kappa}{j}+\left(2N+\frac{\partial u}{\partial x}-v\frac{\partial v}{\partial y}\right)+\left(\frac{\mu_s}{j}\right)\left(\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial y^2}\right), \tag{4}$$

$$\rho\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{k_1}{C_p}\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

$$\rho\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D^*\frac{\partial^2 C}{\partial y^2} \tag{6}$$

where u and v are the velocity components along the x- and y-axis respectively, ρ is the fluid density, μ is the dynamic viscosity, N is the angular or micro rotation velocity, P is the fluid pressure, T and C_p are the fluid temperature and specific heat at constant pressure respectively, C is the species concentration, k_1 and D^* are the thermal conductivity and molecular diffusivity respectively, j is the micro-inertia density, k is a material parameter and $\upsilon_s=(\mu+k/2)\,j$ is the micro rotation viscosity. The appropriate boundary conditions are:

$$\begin{aligned} &y=-h: v=u=0, N=-S\frac{\partial u}{\partial y}\\ &y=+h: v=0, u=\frac{v_0 x}{h}, N=\frac{v_0 x}{h^2}\end{aligned} \tag{7}$$

where S is a boundary parameter and indicates the degree to which the microelements are free to rotate near the channel walls. The case $S = 0$ represents concentrated particle flows in which microelements close to the wall are unable to rotate. Other interesting particular cases that have been considered in the literature include $S = 0.5$ which represents weak concentrations and the vanishing of the antisymmetric part of the stress tensor and $S = 1$ which represents turbulent flow. We introduce the following dimensionless variables [14]:

$$\begin{aligned} &\eta=\frac{y}{h}, \psi=-v_0 x f(\eta), N=\frac{v_0 x}{h^2}g(\eta),\\ &\theta(\eta)=\frac{T-T_2}{T_1-T_2}, \phi(\eta)=\frac{C-C_2}{C_1-C_2}\end{aligned} \tag{8}$$

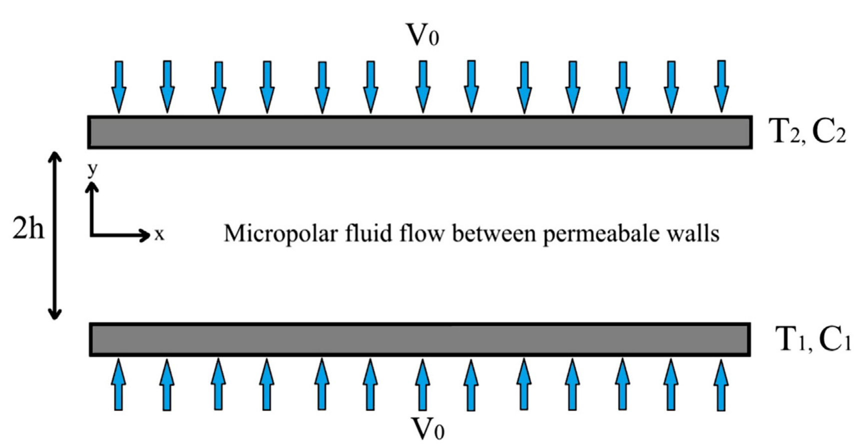

Fig. 1. Geometry of problem.