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An analytical study of unsteady motion of non-spherical particle in plane of Couette flow

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ABSTRACT

An analytical solution of the unsteady two dimensional motion of a non-spherical particle in the plane of Couette flow was acquired using the finite parameter optimal homotopy analysis method. To achieve the best accuracy and ensure the convergence of the results, the averaged residual errors were obtained and minimized. The effects of different initial guesses and the number of convergence-control parameter (k) on the accuracy and efficiency of the problem were studied in detail. It was shown that the current method gives completely reliable results and there is no need to compare the results to those of similar numerical or experimental techniques. Furthermore, the effects of different parameters including sphericity and the proportionality constant on three different base fluids namely: water, ethylene-glycol, and glycerin were investigated. Based on the analytical results, it was shown that non-spherical particles are slower to settle rather than spherical particles and the settling velocity of the particles in the glycerin is much lower than that in the ethylene-glycol and the water base fluids.

1. Introduction

Sedimentation and falling of solid particles in gases and liquids are natural phenomena during many industrial processes [1,2]. The sedimentation refers to the tendency of suspended particles in fluids to settle due to forces acting on them. Primary sedimentation occurs during sediment transport and deposition in pipe lines [3,4], within alluvial channels [5,6], adherence of particles to gas turbine blades [7] and during chemical and powder processes [8,9]. In many of these processes, directing the particles in the fluid is often essential in order to design or improve the process. The majority of the previous studies have considered the steady-state conditions, where the particles achieved their terminal velocity. However, for practical problems, such as raindrops, for measuring the terminal velocity and the viscosity, the falling ball method is used, which requires how the particles reach the settling velocity. There is thus a certain need to consider the unsteady motion of particles, comprehensively.

Several recent studies have investigated the physical significance of some analytical methods, including Homotopy Perturbation Method (HPM) [10,11], and Variational Iteration Method (VIM) [12,13], and their compatibility with physical problems. For instance, Jalaal et al. [14] have used HPM to study the unsteady motion of a spherical particle falling in a Newtonian fluid. Additionally, Torabi and Yaghoobi [15] have implemented Padé approximants to the HPM to find the accelerated motion of a single rigid spherical particle, moving in an incompressible Newtonian media. Meanwhile, Jalaal et al. [16] used an alternative series-based method called the Homotopy Analysis Method (HAM) to solve the nonlinear particle equation of motion whose results are accurate and reliable. In another study, Jalaal et al. [17] presented an HPM solution of a spherical particle in plane of Couette flow. Then, a series of bold works by Yaghoobi and Torabi [18] and Jalaal et al. [17] were done to find the most suitable, efficient, and reliable analytical methods to describe the particle motion in different conditions.

Recently, Hamidi et al. [19] applied the HPM-Padé to solve the coupled equations of a spherical solid particle's motion in plane of Couette flow. Then, Torabi et al. [20] extended their solution with Boubaker polynomial expansion scheme (BPES). However, irregular non-spherical particles are found in most engineering fluids and industrial particulate flows. Recently, Malvandi et al. [21] studied the unsteady motion of a rigid spherical particle in a quiescent shear-thinning power-law fluid by coupling the HPM and the VIM. In this paper, which can be considered as an extension of [19], we have focused on the unsteady motion of a rigid non-spherical particle in a plane of Couette flow. Presenting the governing equations as two coupled nonlinear ordinary differential equations, the analytical *finite-parameter optimal homotopy analysis* solution has been provided which to the best of the authors' knowledge have not been communicated so far. To check the accuracy and the convergence of analytical results, the averaged

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residual errors have been obtained and minimized. It is hoped that the obtained results present useful information for applications as well as complementing the previous studies.

2. Mathematical formulations

Consider the motion of a non-spherical particle in the narrow gap between two infinite horizontal parallel plates filled with a Newtonian fluid. The top plate is moving at speed $U = U_0$ and the bottom plate is kept stationary which is known as "Couette flow". The geometry of the problem is shown in Fig. 1. The particle will rotate with a constant angular velocity given by one-half of the curl of the fluid motion. The direction of rotation is clockwise as seen by an observer for whom the flow is from left to right [22]. In addition, the gravity and the buoyancy effects will be assumed negligible. In fact, the combined effects of inertia, drag, and lift forces determined the particle's motion. The lift force includes two components: one due to particle's rotation, and the other due to the shear stresses, as explained by Rubinow and Keller [23] and Saffman [24], respectively. Thus, the lift forces of particles due to the rotation and shear stresses may be obtained as follows, respectively:

$$\text{due to rotation} \quad F_{L1} = \frac{1}{2}\pi a^3 \rho\alpha(\dot{y}, \alpha y - \dot{x}, 0) \tag{1}$$

and,

$$\text{due to shear} \quad F_{L2} = 6.46\rho a^2 \sqrt{\alpha\upsilon}(0, \alpha y - \dot{x}, 0) \tag{2}$$

where $a, \rho, \upsilon, \dot{x}$, and $\dot{y}$ are the particle equivalent volume radius, density of the fluid, kinematic fluid viscosity and particle's velocity in the x- and y-directions, respectively. $\alpha = \frac{U_0}{h}$ is a positive proportionality constant with the dimension of (1/t) in which h is the distance between the plates. The inertia force is simply the mass of the particle times its acceleration $(\ddot{x}, \ddot{y}, 0)$:

$$F_I = m_p(\ddot{x}, \ddot{y}, 0) \tag{3}$$

where m_p signifies the particle mass. Furthermore, the added-mass effect which is due to the acceleration of fluid around the particle could be written as follows:

$$F_v = m_v(\ddot{x}, \ddot{y}, 0) = \frac{2}{3}\pi a^3 \rho(\ddot{x}, \ddot{y}, 0). \tag{4}$$

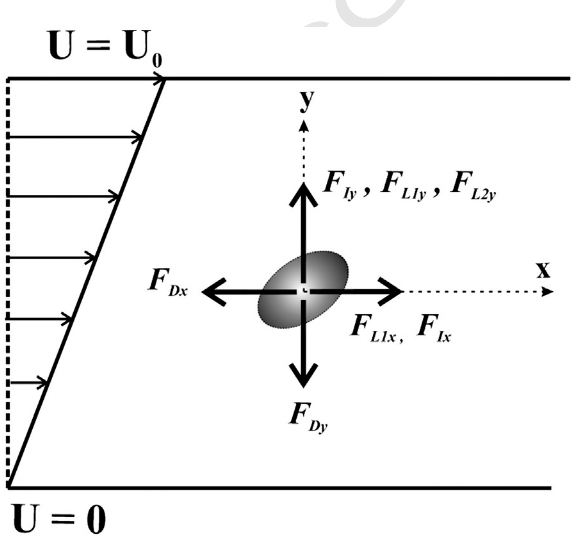

Fig. 1. Schematic illustration of the physical model.

Calculating the drag force requires choosing an adaptable drag coefficient. The proper formulation of the drag coefficient in a wide variety of Reynolds numbers, shapes, and sphericities is the main issue in this problem. Up to now, several correlations in terms of Reynolds number Re and sphericity ϕ have been reported in the literature [18]. Here, Chien's correlation [25], valid in the ranges of $0.01 < \text{Re} < 10000$ and $0.2 < \phi < 1$ for different shapes of particle is used, which can be expressed as

$$C_D = \frac{30}{\text{Re}} + 67.289e^{-5.03\phi}. \tag{5}$$

The relative velocities of the fluid and particle are obtained as $U_r = (\dot{x} - \alpha y, \dot{y}, 0)$; so, Eq. (6) is defined to determine the drag force:

$$F_D = \frac{1}{2}\pi a^2 \rho U_r^2 C_D = 7.5\pi a\mu(\dot{x} - \alpha y, \dot{y}, 0) + 33.6445e^{(-5.03)\phi}\pi a^2 \rho\left((\dot{x} - \alpha y)^2, (\dot{y})^2, 0\right) \tag{6}$$

where μ is the fluid viscosity. The equations of motion of particle have been obtained by the second-law Newtonian formula in the x- and y-directions:

$$F_I + F_v = F_{L1} + F_{L2} - F_D. \tag{7}$$

Therefore, the governing equations are determined as:

$$\left(m_p + m_v\right)\ddot{x} = \frac{1}{2}\pi a^3 \rho\alpha\,\dot{y} - \left(7.5\pi a\mu(\dot{x} - \alpha y) + 33.6445e^{(-5.03)\phi}\pi a^2 \rho(\dot{x} - \alpha y)^2\right) \tag{8}$$

$$\left(m_p + m_v\right)\ddot{y} = \left(\frac{1}{2}\pi a^3 \rho\alpha + 6.46\rho a^2 \sqrt{\alpha\upsilon}\right) \times (\alpha y - \dot{x}) - \left(7.5\pi a\mu\,\dot{y} + 33.6445e^{(-5.03)\phi}\pi a^2 \rho\dot{y}^2\right). \tag{9}$$

Eqs. (8) and (9) can be abbreviated in the following forms:

$$\ddot{x} - A\,\dot{y} + B(\dot{x} - \alpha y) + D(\dot{x} - \alpha y)^2 = 0 \tag{10}$$

$$\ddot{y} + B\,\dot{y} + D\dot{y}^2 + (A + C)(\dot{x} - \alpha y) = 0 \tag{11}$$

where the coefficients A–D are defined as:

$$\begin{aligned} A &= \frac{1}{2\left(m_p + m_v\right)}\pi a^3 \rho\alpha \ : [1/s], \quad B = \frac{7.5\pi a\mu}{\left(m_p + m_v\right)} \ : [1/s] \\ C &= \frac{6.46\rho a^2 \sqrt{\alpha\upsilon}}{\left(m_p + m_v\right)} \ : [1/s], \quad D = \frac{33.6445e^{(-5.03)\phi}\pi a^2 \rho}{\left(m_p + m_v\right)} \ : [1/m]. \end{aligned} \tag{12}$$

To obtain a non-trivial solution, it is essential to specify the non-zero initial conditions. The following might represent either injection of the particle into the fluid or statistical fluctuations:

$$\begin{aligned} x(t = 0) &= 0, \quad \dot{x}\,(t = 0) = u_0 \\ y(t = 0) &= 0, \quad \dot{y}\,(t = 0) = v_0. \end{aligned} \tag{13}$$

It should be stated that the order of the ODE system Eqs. (10)–(11) can be reduced twice by choosing $\dot{x} - ay$ and $\dot{y}$ as new independent variables. However, in view of showing the strength of our analytical method, we leave this transformation and the results will be obtained for x and y.

3. Analytical approximations

Liao [26–28] introduced Homotopy Analysis Method (HAM) to obtain analytical approximations of strongly nonlinear differential

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