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Micropolar fluid flow and heat transfer in a permeable channel using analytical method

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ABSTRACT

In this study, micropolar fluid flow in a channel subject to a chemical reaction is investigated analytically using homotopy perturbation method (HPM). The concept of homotopy perturbation method is briefly introduced and employed to derive solutions of nonlinear equations. The obtained results from HPM are compared with those of obtained from numerical method (four-order Runge–Kutta method) to verify the accuracy of the proposed method. The results reveal that the HPM can achieve suitable results in predicting the solution of such problems. The effects of significant parameters such as Reynolds number, micro rotation/angular velocity and Peclet number on the flow, heat transfer and concentration characteristics are discussed. For both suction and injection it can be found that Reynolds number and Peclet number have direct relationship with Nusselt number and Sherwood number.

1. Introduction

The theory of a micropolar fluid derives from the need to model the flow of fluids that contain rotating micro-constituents. A micropolar fluid is the fluid with internal structures in which coupling between the spin of each particle and the macroscopic velocity field is taken into account. It is a hydro dynamical framework suitable for granular systems which consist of particles with macroscopic size. Eringen [1] was the first pioneer of formulating the theory of micropolar fluids. His theory introduces new material parameters, an additional independent vector field – the microrotation – and new constitutive equations which must be solved simultaneously with the usual equations for Newtonian fluid flow. The field of micropolar fluids is very rich in literature, with various aspects of the problem having been investigated. Examples include Gorla [2], Rees and Bassom [3] who investigated the flow of a micropolar fluid over a flat plate and Kelson and Desseaux [4], who studied flow of micropolar fluids on stretching surfaces. Heat and mass transfer are important in many industrial and technological processes. In manufacturing and metallurgical processes, heat and mass transfer occur simultaneously. The influence of a chemical reaction and thermal radiation on the heat and mass transfer in MHD micropolar flow over a vertical moving plate in a porous medium with heat generation was studied by Mohamed and Abo-Dahab [5]. Recently effect of using micropolar fluid, nanofluid, etc. on flow and heat transfer has been studied by several authors [6–15].

Most of engineering problems, especially some heat transfer equations are nonlinear, therefore some of them are solved using numerical solution and some are solved using the different analytic method, such as perturbation method, homotopy perturbation method, variational iteration method introduced by He [16]. Therefore, many different methods have recently introduced some ways to eliminate the small parameter. One of the semi-exact methods which do not need small parameters is the homotopy perturbation method (HPM). HPM, proposed first by He in 1998 and was further developed and improved by He [17]. The method yields a very rapid convergence of the solution series in the most cases. The method yields a very rapid convergence of the solution series in the most cases. The HPM proved its capability to solve a large class of nonlinear problems efficiently, accurately and easily with approximations convergency very rapidly to solution. Usually, few iterations lead to high accuracy solution. Heat transfer of a nanofluid flow which is squeezed between parallel plates was investigated analytically using HPM by Sheikholeslami and Ganji [18]. They reported that Nusselt number has direct relationship with nanoparticle volume fraction, the squeeze number and Eckert number when two plates are separated but it has reverse relationship with the squeeze number when two plates are squeezed. This method is employed for many researches in engineering sciences. He's homotopy perturbation method is applied to obtain approximate analytical solutions for the motion of a spherical particle in a plane couette flow by Jalal et al.

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Nomenclature

C	species concentration
D^*	thermal conductivity and molecular diffusivity
f	dimensionless stream function
g	dimensionless micro rotation
h	width of channel
HPM	homotopy perturbation method
j	micro-inertia density
N	micro rotation/angular velocity
$N_{1,2,3}$	dimensionless parameter
Nu	Nusselt number
Sh	Sherwood number
Sc	Schmidt number
p	pressure
Pr	Prandtl number
Pe	Peclet number
q	mass transfer parameter
Re	Reynolds number
T	fluid temperature
s	micro rotation boundary condition
(u,v)	Cartesian velocity components
(x,y)	Cartesian coordinate components parallel & normal to channel axis, respectively

Greek symbols

η	similarity variable
θ	dimensionless temperature
μ	dynamic viscosity
κ	coupling coefficient
ρ	fluid density
υ_s	micro rotation/spin-gradient viscosity
ψ	stream function

[19]. Sheikholeslami et al. [20] studied the problem of laminar nanofluid flow in a semi-porous channel. They found that the velocity boundary layer thickness decreases with increasing Reynolds number and nanoparticle volume fraction, and it increases while Hartmann number increases. Sheikholeslami et al. [21] analyzed the magnetohydrodynamic nanofluid flow and heat transfer between two horizontal plates in a rotating system. Their results indicated that, for both suction and injection, Nusselt number has a direct relationship with nanoparticle volume fraction. In recent years some researchers used new methods to solve these kinds of problems [22–37].

In this study, we have applied HPM to find the approximate solutions of nonlinear differential equations governing the micropolar fluid flow in a channel. A comparison between the results and the numerical solution has been provided. The velocity, temperature and concentration profiles are shown and the influence of Reynolds numbers, micro rotation/angular velocity and Peclet number on the flow, heat transfer and concentration characteristics are discussed in detail.

2. Mathematical formulation

We consider the steady laminar flow of a micropolar fluid along a two-dimensional channel with porous walls through which fluid is uniformly injected or removed with speed v_0. The lower channel wall has a solute concentration C_1 and temperature T_1 while the upper wall has solute concentration C_2 and temperature T_2 as shown in Fig. 1. Using Cartesian coordinates, the channel walls are parallel to the x-axis and

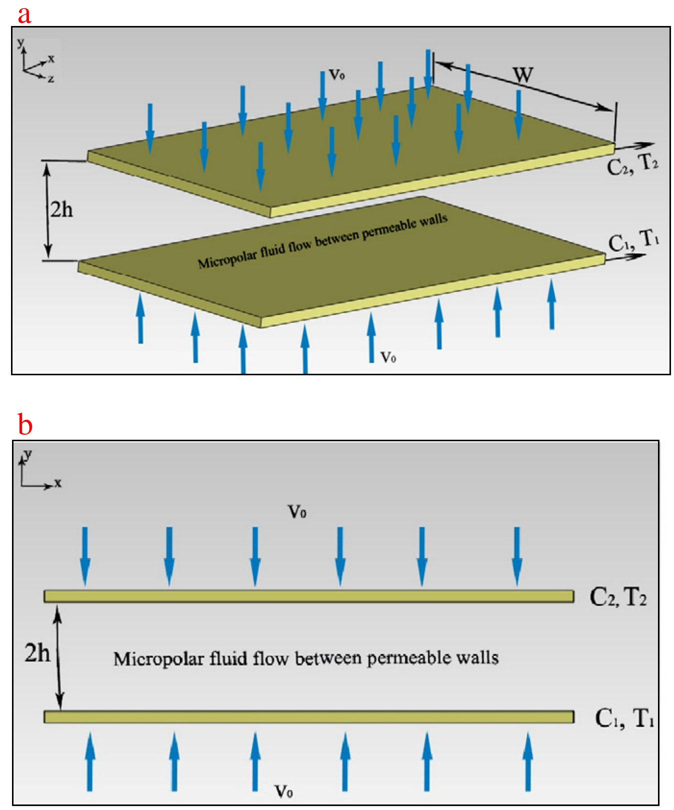

Fig. 1. (a) Geometry of problem; (b) x–y view at $z = W/2$.

located at $y = \pm h$ where $2h$ is the channel width. The relevant equations governing the flow are [38]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+(\mu+\kappa)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\kappa\frac{\partial N}{\partial y}, \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+(\mu+\kappa)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\kappa\frac{\partial N}{\partial x}, \tag{3}$$

$$\rho\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=-\frac{\kappa}{j}\left(2N+\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)+\left(\frac{\mu_s}{j}\right)\left(\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial y^2}\right), \tag{4}$$

$$\rho\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{k_1}{c_p}\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

$$\rho\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D^*\frac{\partial^2 C}{\partial y^2}. \tag{6}$$

where u and v are the velocity components along the x- and y-axes respectively, ρ is the fluid density, μ is the dynamic viscosity, N is the angular or micro rotation velocity, P is the fluid pressure, T and c_p are the fluid temperature and specific heat at constant pressure respectively, C is the species concentration, k_1 and D^* are the thermal conductivity and molecular diffusivity respectively, j is the micro-inertia density, k is a material parameter and $\upsilon_s=\left(\mu+\frac{k}{2}\right)j$ is the micro rotation viscosity.