Journal of Molecular Liquids 194 (2014) 41–47

Contents lists available at ScienceDirect

Journal of Molecular Liquids

journal homepage: www.elsevier.com/locate/molliq

On model for three-dimensional flow of nanofluid: An application to solar energy

Junaid Ahmad Khan [a], M. Mustafa [d], T. Hayat [b,c,*], M. Asif Farooq [d], A. Alsaedi [c], S.J. Liao [e]

[a] Research Centre for Modeling and Simulation (RCMS), National University of Sciences and Technology (NUST), Islamabad 44000, Pakistan
[b] Department of Mathematics, Quaid-I-Azam University, 45320, Islamabad 44000, Pakistan
[c] Department of Mathematics, Faculty of Science, King Abdulaziz University, P. O. Box 80257, Jeddah 21589, Saudi Arabia
[d] School of Natural Sciences, National University of Sciences and Technology (NUST), Islamabad 44000, Pakistan
[e] Department of Mathematics & State Key Lab of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, China

ARTICLE INFO

Article history:
Received 26 November 2013
Accepted 24 December 2013
Available online 24 January 2014

Keywords:
Three-dimensional flow
Nanofluid
Bi-directional stretching sheet
Shooting method
Convective boundary conditions

ABSTRACT

Laminar three-dimensional flow of nanofluid over a bi-directional stretching sheet is investigated. Convective boundary conditions are used for the analysis of thermal boundary layer. Mathematical model containing the combined effects of Brownian motion and thermophoretic diffusion of nanoparticles is adopted. The formulated differential system is solved numerically using a shooting method with fourth–fifth-order Runge–Kutta integration technique. The solutions depend on various interesting parameters including velocity ratio parameter (λ), Brownian motion parameter (N_b), thermophoresis parameter (N_t), Prandtl number (Pr), Lewis number (Le) and the Biot number (γ). It is noticed that fields are largely influenced with the variations of these parameters. The results are compared with the existing studies for the two-dimensional flows and found in an excellent agreement. The study reveals that nanoparticles in the base fluid offer a potential in improving the convective heat transfer performance of various liquids.

1. Introduction

The nanofluids in view of the extraordinary thermal conductivity enhancement have been recognized useful in several industrial and engineering applications. One of the technological applications of nanoparticles that hold enormous promise is the use of heat transfer fluids containing suspensions of nanoparticles to confront cooling problems in the thermal systems. Solar power is a direct way of obtaining heat, water and electricity from the nature. Researchers concluded that heat transfer and solar collection processes can be improved through the addition of nanoparticles in the fluids. Use of nanofluids as coolants would allow for smaller size and better positioning of the radiators which eventually consumes less energy for overcoming resistance on the road. Nanoparticles in refrigerant/lubricant mixtures could enable a cost effective technology for improving the efficiency of chillers that cool large buildings. Also the classical heat transfer fluids such as ethylene glycol, water and engine oil have limited heat transfer capabilities due to their low thermal conductivity and thus cannot congregate with modern cooling requirements. On the other hand thermal conductivity of metals is extremely higher in comparison to the conventional heat transfer fluids. Masuda et al. [1] explored the variations in the thermal conductivities and viscosities of liquids through the dispersion of ultra-fine particles in the base fluids. Choi and Eastman [2] combined the conventional heat transfer fluids with nanometer sized metallic particles and observed a significant increase in the thermal conductivity of the resulting liquid which was termed as nanofluid. In another paper, Eastman et al. [3] discussed an abnormal increase in the thermal conductivity of ethylene glycol based nanofluids.

Buongiorno [4] studied the convective transport phenomena in nanofluids and concluded that out of the seven slip mechanisms only Brownian motion and thermophoretic diffusion of nanoparticles contribute to the massive increase in the absolute thermal conductivity of the liquids. He also developed a mathematical model for nanofluid flow which incorporates the simultaneous effects of Brownian motion and thermophoretic diffusion of nanoparticles. Kuznetsov and Nield [5] investigated the natural convective boundary-layer flow of a nanofluid past a vertical flat plate using Buongiorno's model. Cheng–Minkowycz problem for natural convection flow of nanofluid past a vertical plate embedded in a porous medium was studied by Nield and Kuznetsov [6]. The two-dimensional flow of nanofluid over a linearly stretching sheet was conducted by Khan and Pop [7]. They computed the numerical solutions of the developed differential system by Keller-box and provided a detailed analysis of Brownian motion and thermophoresis effects on the heat transfer characteristics. Makinde and Aziz [8] extended this work by considering convective boundary conditions. They showed that strength of convective heating has a significant impact on the thermal boundary layer. Rana and Bhargava [9] provided finite element solutions for two-dimensional flow of

---

* Corresponding author at: Department of Mathematics, Quaid-I-Azam University, 45320, Islamabad 44000, Pakistan. Tel.: +92 51 90642172.
E-mail address: pensy_t@yahoo.com (T. Hayat).

0167-7322/$ – see front matter © 2014 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.molliq.2013.12.045

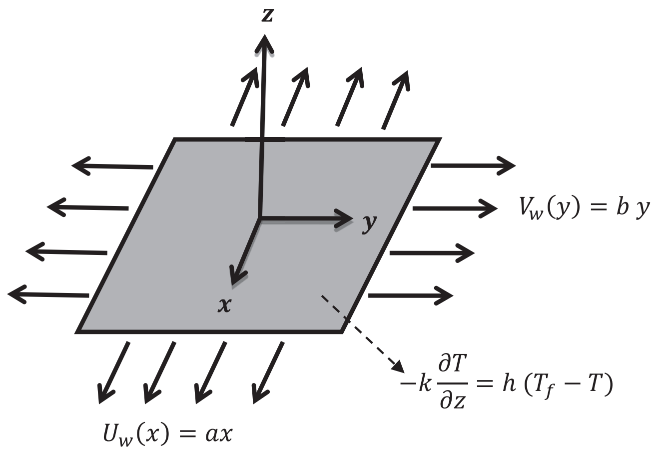

Fig. 1. Physical configuration and coordinate system.

nanofluid over a nonlinearly stretching sheet. Mustafa et al. [10] discussed stagnation-point flow of nanofluid towards a stretching surface by homotopy analysis method (HAM). In another paper Mustafa et al. [11] investigated the unsteady boundary layer flow of nanofluid past a stretching sheet. Flow of an electrically conducting nanofluid past a stretching cylinder was analyzed by Ashorynejad et al. [12]. Stagnation-point flow of nanofluid over a linearly stretching/shrinking surface was considered by Yacob et al. [13]. They have shown the existence of dual solutions in the case of shrinking sheet. Effect of internal heat generation on the nanofluid flow over a permeable stretching sheet was addressed by Hamad and Ferdows [14]. Free convective flow of nanofluid past a vertical flat surface with Newtonian heating boundary conditions was explored by Uddin et al. [15]. Khan and Pop [16] numerically investigated the forced convection flow of nanofluid through a porous medium using Brinkman–Forchheimer model. Numerical and analytic solutions for stagnation-point flow of nanofluid past an exponentially stretching sheet were provided by Mustafa et al. [17]. Stagnation-point flow of nanofluid past flat vertical plate with convective boundary conditions was examined by Makinde et al. [18]. MHD natural convection flow of nanofluid in a cavity is addressed by Sheikholeslami et al. [19]. Turkyilmazoglu and Pop [20] analyzed unsteady natural convection flow of nanofluid past a vertical infinite plate. MHD flow of nanofluid due to a rotating disk is studied by Rashidi et al. [21]. Nadeem and Haq [22] explored the MHD boundary layer flow over a permeable stretching sheet in the presence of nanoparticles. Thermal radiation and viscous dissipation effects on the unsteady boundary layer flow of nanofluid over a stretching sheet were presented by Khan et al. [23]. Effect of solar energy radiation on the unsteady boundary layer flow of nanofluid past a wedge was discussed by Mohamad et al. [24]. They concluded that presence of nanoparticles in the base fluids allows deeper penetration of radiations. Sheikholeslami et al. [25] carried out an investigation to provide an application of LBM in simulation of natural convection nanofluid. The fluid fills the square cavity which has curve boundaries. Unsteady squeezing flow of nanofluid by ADM was investigated by Sheikholeslami et al. [26]. Turkyilmazoglu [27] provided both exact and analytical solutions for hydromagnetic flow of nanofluid with slip condition. In another investigation, Turkyilmazoglu [28] discussed the unsteady flow of nanofluid passing through a vertical plate.

The study of heat transfer in the boundary layer flows due to stationary or moving surface has relevance in various industrial applications. The seminal work on the laminar boundary flow over a flat plate at zero incidence in a quiescent ambient fluid with uniform free stream was reported by Blasius [29]. He provided an analytic solution of the problem in the power series form. The numerical solution to the Blasius problem was computed by Howarth [30]. In contrast to [29], the flow over a continuously moving plate was considered by Sakiadis [31]. Crane [32] extended this idea for a stretching sheet and provided an exact solution for the velocity distribution. The flow over a stretching sheet is involved in the extrusion process, fabrication of plastic, rubber and metallic sheets, glass and fiber production, wire drawing, hot rolling, melt spinning, transportation etc. In view of such applications, the Crane's problem has been extensively considered by the researchers even for the three-dimensional flows over a bi-directional stretching sheet. The seminal research in this direction was conducted by Wang [33]. He had also shown that classical problems of two-dimensional and axisymmetric flows due to stretching sheet can be easily recovered from his work. Unsteady three-dimensional flow past an impulsively stretching surface was analyzed by Lakshmisha et al. [34]. Series solutions for three-dimensional flow of an electrically conducting viscous fluid were provided by Xu et al. [35]. Sajid et al. [36] also derived analytic solutions for three-dimensional flow of elasto-viscous fluid over a stretching sheet. Liu and Andersson [37] numerically investigated the heat transfer characteristics over a bi-directional stretching sheet with variable wall temperature. Laminar three-dimensional flow of viscous fluid filling a porous space with heat and mass transfer was addressed by Hayat et al. [38]. The analytic solutions for three-dimensional flows of non-Newtonian fluids over a stretching sheet have been reported by Hayat et al. [39,40]. Recently Liu et al. [41] provided an interesting

Table 1
Comparison of values of reduced Nusselt number $-\theta'(0)$ with the previous studies when $N_b = N_t = 0$, $\lambda = 0$, $\gamma = 1000$.

Pr	Khan and Pop [7]	Makinde and Aziz [8]	Gorla and Sidawi [42]	Present (bvp4c)	Present (shooting method)
0.07	0.0663	0.0656	0.0656	0.06562	0.06562
0.20	0.1691	0.1691	0.1691	0.16909	0.16909
0.70	0.4539	0.4539	0.5349	0.45392	0.45392
2.00	0.9113	0.9114	0.9114	0.91136	0.91136
7.00	1.8954	1.8954	1.8905	1.89542	1.89542
20.00	3.3539	3.3539	3.3539	3.35394	3.35391
70.00	6.4621	6.4622	6.4622	6.46231	6.46220

Table 2
Comparison of values of the reduced Nusselt number $-\theta'(0)$ with Makinde and Aziz [8] with $Le = \Pr = 10$, $\gamma = 0.1$, $\lambda = 0$.

N_t	*Nur* ($N_b = 0.1$)	*Nur* ($N_b = 0.2$)	*Nur* ($N_b = 0.3$)	*Nur* ($N_b = 0.4$)	*Nur* ($N_b = 0.5$)
0.1	0.092907 (0.0929)	0.087332 (0.0873)	0.076878 (0.0769)	0.059665 (0.0597)	0.038325 (0.0383)
0.2	0.092732 (0.0927)	0.086762 (0.0868)	0.075082 (0.0751)	0.055349 (0.0553)	0.032498 (0.0325)
0.3	0.092545 (0.0925)	0.086119 (0.0861)	0.072917 (0.0729)	0.050269 (0.0503)	0.026905 (0.0269)
0.4	0.092344 (0.0923)	0.085385 (0.0854)	0.070265 (0.0703)	0.044558 (0.0445)	0.022010 (0.0220)
0.5	0.092126 (0.0921)	0.084538 (0.0845)	0.066974 (0.0700)	0.038620 (0.0386)	0.018035 (0.0180)