



# An analytical study on unsteady motion of vertically falling spherical particles in quiescent power-law shear-thinning fluids

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## ABSTRACT

Unsteady motion of a rigid spherical particle in a quiescent shear-thinning power-law fluid was investigated analytically. The accurate series solution was found by coupling the homotopy-perturbation method (HPM) and the variational iteration method (VIM). The results were compared with those obtained from VIM and the established finite difference scheme. It was shown that both methods (VIM and HPM–VIM) gave accurate results; however, the amount of calculations required for HPM–VIM was significantly reduced. In addition to improved efficiency, it was revealed that HPM–VIM leads to completely reliable and precise results. The terminal settling velocity—that is the velocity at which the net forces on a falling particle eliminate—for three different spherical particles (made of plastic, glass and steel) and three flow behavior index  $n$ , in two sets of power-law non-Newtonian fluids was investigated, based on the series solution. Analytical results obtained indicated that the time of reaching the terminal velocity in a falling procedure is significantly declined with growing the particle size. Further, with approaching flow behavior to Newtonian behavior from shear-thinning properties of flow ( $n \rightarrow 1$ ), the transient time to achieving the terminal settling velocity is decreased.

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## 1. Introduction

The sedimentation and falling of solid particles in gases and liquids is a natural phenomenon that occurs in many industrial processes. Primarily, sedimentation results from a tendency of suspended particles in fluids to settle and come to rest, due to the forces acting on them through the fluid [1]. Common examples include separation of liquid–solid mixtures, sprays and atomization, sediment transportation and deposition in pipe lines [2,3], alluvial channels [4,5], and chemical and powder processing. In many processes it is often essential to obtain the route of particles that accelerates in the fluid region for designing or improving the processes. The majority of previous studies have considered the steady-state conditions and where the particles achieved their terminal velocity. Also, several works have been done to study the unsteady motion of particles in Newtonian fluids [6–11] due to its applications in classification, centrifugal collection and separation (some of the unit operations which require the trajectories of particles accelerating in fluid). Further, the distance required to reach the terminal velocity is necessary for viscosity measurements of fluid with the falling ball experiment.

Along with the same proposition, many researchers realized the physical significance of some analytical methods such as the homotopy

perturbation method (HPM) [12], variational iteration method (VIM) [13,14] and homotopy analysis method [15] and its compatibility with the physical problems as the unsteady motion of spherical particles in Newtonian fluids. These methods were originally proposed by He [16,17] to achieve the series solution of strongly nonlinear differential equations. Jalaal et al. [18] used HPM to study the unsteady motion of a spherical particle falling in a Newtonian fluid for a range of Reynolds number to obtain a solution for nonlinear equations of a falling spherical with drag coefficient. Then, Jalaal et al. [19] used a series-based method called homotopy analysis method (HAM) in order to solve nonlinear particle equation of motion whose results are very accurate and reliable. Meanwhile, an unsteady rolling motion of spheres in inclined tubes filled with incompressible Newtonian fluids was conducted by Jalaal and Ganji [19]. Later, Hamidi et al. [20] applied the HPM–Padé to solve the coupled equations of a spherical solid particle's motion in Couette flow. They showed that using of the Padé approximation can enhance the convergence of the homotopy series solution obtained by HPM.

Majority of the abovementioned studies has described the motion of solid particles in Newtonian suspensions only, however, many slurries and concentrated suspensions, which are treated in the materials processing industry, behave as non-Newtonian liquids and proper consideration has to be made [21–26]. The numerical solution of Bagchi and Chhabra [27] is one of the studies in this field. They reported the distance traveled by accelerating spherical particles in downward vertical motion of particles in power law liquids. However, to the best of the authors' knowledge, no attempt has thus far been communicated in

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order to find the semi-analytical solution of the accelerated motion of spherical particles in shear-thinning fluids. Therefore, a combination of HPM and VIM [28] was used to find efficient, reliable and precise series solutions. In order to consider the non-Newtonian fluid flow, the power-law model was employed. Furthermore, the terminal settling velocity for three rigid spherical particles namely plastic, glass and steel, vertically falling in the quiescent power-law fluids was determined. In terms of obtaining the best accuracy of the analytical results, a comparison was made by a numerical solution via finite difference scheme.

## 2. Problem formulations

Consider the one-dimensional accelerated motion of a rigid spherical particle vertically falling to an infinite extent of a power-law shear-thinning fluid as shown in Fig. 1. The forces acting on a falling body are usually gravity, buoyancy, inertia, Basset history force, virtual mass and drag force. From the Lagrangian viewpoint, the dynamic of particles submerged in a fluid could be obtained by integrating the forces balanced on them. According to the studies of Renganathan et al. [29] and Bagchi and Chhabra [27], the Basset force can be assumed to be negligible when the density of the spherical particle is much larger than that of the liquid. Under this condition, the equation of motion describing the falling motion of the particle can be written as [19],

$$m \frac{du}{dt} = mg \left(1 - \frac{\rho}{\rho_s}\right) - \frac{\pi D^2 \rho C_D}{8} u^2 - \frac{\pi D^3 \rho}{12} \frac{du}{dt} \quad (1)$$

where  $D$ ,  $m$ ,  $\rho_s$  and  $C_D$  are the particle diameter, particle mass, particle density and drag coefficient, respectively. From left to right, the terms

represent inertia, gravity–buoyancy, drag and virtual mass (added mass effect due to acceleration of fluid around the particle). The complexity of the above equation arises from the strong non-linear nature of the drag coefficient. The proper formulation of the drag coefficient has routinely been obtained by numerical or experimental results. It is well known today that the drag coefficient for a sphere in a power-law fluid could be expressed as

$$C_D = f(\text{Re}, n). \quad (2)$$

For a creeping flow region ( $\text{Re} \ll 1$ ), the drag coefficient could be obtained from Stokes law in the following form

$$C_D = \frac{24}{\text{Re}} X(n) \quad (3)$$

where  $\text{Re} = \rho u^2 - n D^n / K$  is the Reynolds number.  $n$  and  $K$  are the flow behavior index and consistency coefficient, respectively.  $X(n)$ , a deviation factor, was obtained by researchers via numerical or experimental results. Here, a well-correlated equation of Renaud et al. [30] was used as follows,

$$X(n) = 6^{(n-1)/2} \left( \frac{3}{n^2 + n + 1} \right)^{n+1}. \quad (4)$$

The correlated equation is valid for both shear-thinning ( $n < 1$ ) and shear-thickening ( $n > 1$ ) fluid behavior. By substituting Eqs. (3) and (4) into Eq. (1) and by rearranging the parameters one could give

$$a \frac{du}{dt} + b(n)u^n - d = 0, \quad u(0) = v_0 \quad (5)$$

in which

$$a = m + \frac{1}{12} \pi D^3 \rho, \quad b(n) = 3\pi K X(n) D^{2-n}, \quad d = mg \left(1 - \frac{\rho}{\rho_s}\right). \quad (6)$$

Eq. (5) is classified an IVP (initial value problem) differential equation, which could be solved with suitable numerical methods such as the finite difference scheme. The numerical solution of the problem is not within the scope of this paper, but the analytical solution is described in the following section.

## 3. Mathematical methods

Before presenting the results, it is necessary to provide some background knowledge about the mathematical methods employed. Therefore, in this section, some basic relationships and theories concerning VIM and the combination of HPM and VIM are presented.

### 3.1. Variational iteration method – VIM

VIM is an iterative-integral-based scheme proposed by He [31–33] which has been successfully employed by numerous authors in a wide variety of linear and nonlinear problems [34–39]. The method is based on constructing a correction functional using a general Lagrange multiplier, with the multiplier chosen in such a way that its correction solution is improved with respect to the initial approximation, or to the trial function. The non-linear differential equation is considered to explain the fundamental initiative in the following general form:

$$Lu(t) + Nu(t) = g(t) \quad (7)$$

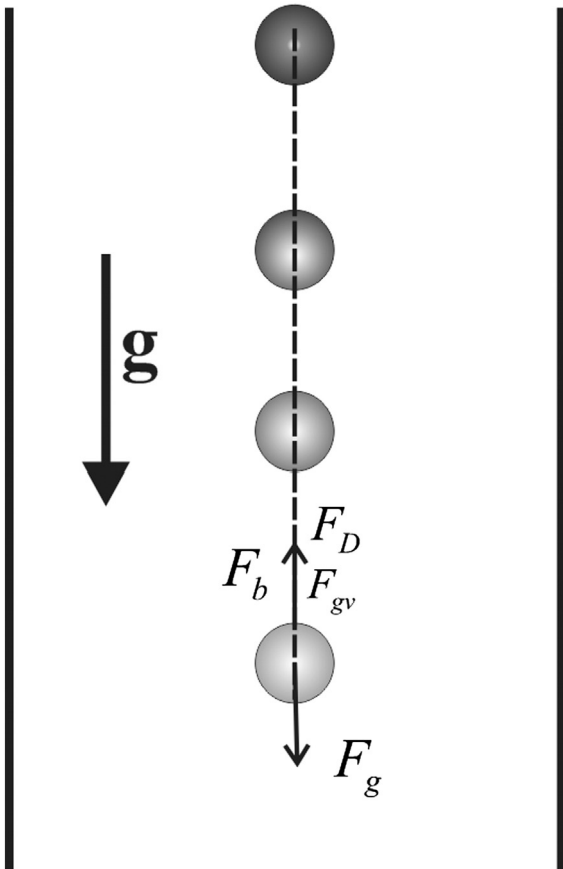


Fig. 1. Geometry of physical model.

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