Journal of Molecular Liquids 190 (2014) 159–168

Contents lists available at ScienceDirect

Journal of Molecular Liquids

journal homepage: www.elsevier.com/locate/molliq

Heat transfer and nanofluid flow in suction and blowing process between parallel disks in presence of variable magnetic field

M. Hatami [a,b,*], D.D. Ganji [b]

[a] Esfarayen University, Engineering and Technical College, Mechanical Engineering Department, Esfarayen, North Khorasan, Iran
[b] Babol University of Technology, Mechanical Engineering Faculty, Energy Conversion Department, Babol, Mazandaran, Iran

ARTICLE INFO

Article history:
Received 15 October 2013
Received in revised form 7 November 2013
Accepted 9 November 2013
Available online 20 November 2013

Keywords:
Nanofluid
LSM
Squeezing flow
Nusselt number
Sherwood number

ABSTRACT

In this paper, nanofluid flow and heat transfer analysis between two parallel disks are investigated using Least Square Method (LSM) and numerical method. A variable magnetic field is applied to the lower stationary disk and the upper disk can move towards or away from the lower disk. After obtaining the governing equations and solving them by LSM, the accuracy of results is examined by fourth order Runge–Kutta numerical method, then the influence of the Squeeze number (S), Hartmann number (M), Brownian motion parameter (*Nb*), thermophrotic parameter (*Nt*) and Prandtl number (*Pr*) on Nusselt number (*Nu*), Sherwood number (*Shr*), non-dimensional temperature, velocity and nanoparticle concentration are investigated. The results indicated that by increasing the *Nb* and *Nt*, Nusselt number increases, but Sherwood number decreases with *Nt* and increases with *Nb*.

1. Introduction

The study of heat and mass transfer unsteady squeezing viscous flow between two parallel plates in motion normal to their own surfaces independent of each other and arbitrary with respect to time has been regarded as one of the most important research topics due to its wide spectrum of scientific and engineering applications such as hydrodynamical machines, polymer processing, lubrication system, chemical processing equipment, formation and dispersion of fog, damage of crops due to freezing, food processing and cooling towers. The first work on the squeezing flow under lubrication approximation was reported by Stefan [1]. After that, Mahmood et al. [2] investigated the heat transfer characteristics in the squeezed flow over a porous surface. Aziz [3] considered the outcome of time-dependent chemical reaction on the flow of a viscous fluid past an unsteady stretching sheet. Also, magnetohydrodynamic squeezing flow of a viscous fluid between parallel disks was analyzed by Domairry and Aziz [4]. Mustafa et al. [5] solved the problem of the fluid flow between parallel plates by Homotopy Analysis Method (HAM [6–10]) and Joneidi et al. [11] provided the analytic and numerical solutions for magnetohydrodynamic squeezing flow between parallel disks. Squeezing flow of second grade fluid between parallel disks (one of which is porous) has been analyzed by Hayat et al. [12] and heat transfer characteristics in the unidirectional squeezing flow between parallel disks have been studied by Duwairi et al. [13]. Furthermore, Hashmi et al. [14] investigated the magnetohydrodynamic squeezing flow of nanofluid between parallel disks by HAM. In another analytical study, heat transfer of a nanofluid flow which is squeezed between parallel plates was investigated using Homotopy perturbation method (HPM) by Sheikholeslami and Ganji [15]. They reported that Nusselt number has direct relationship with nanoparticle volume fraction, the squeeze number and Eckert number when two plates are separated but it has reverse relationship with the squeeze number when two plates are squeezed. For more information, some of studies in the nanofluid studies can be found in Refs. [16–22].

There are some simple and accurate approximation techniques for solving nonlinear differential equations called the Weighted Residuals Methods (WRMs). Collocation, Galerkin and Least Square Method (LSM) are examples of the WRMs which are introduced by Ozisik [23] for using in heat transfer problem. Stern and Rasmussen [24] used collocation method for solving a third order linear differential equation. Vaferi et al. [25] have studied the feasibility of applying of Orthogonal Collocation method to solve diffusivity equation in the radial transient flow system. Recently Hatami et al. [26] used LSM for heat transfer study through porous fins also the problem of laminar nanofluid flow in a semi-porous channel in the presence of transverse magnetic field is investigated analytically by Sheikholeslami et al. [27] using LSM. Furthermore LSM is introduced by Aziz and Bouaziz [28,29] for predicting the performance of longitudinal fins. They found that least squares method is simple compared with other analytical methods. Shaoqin and Huoyuan [30] developed and analyzed least-squares approximations for the incompressible magneto-hydrodynamic equations. Hatami and Ganji [31] found that LSM is more appropriate than other analytical

* Corresponding author at: Esfarayen University, Iran. Tel.: +98 915 718 6374.
E-mail addresses: m.hatami2010@gmail.com (M. Hatami), ddg_davood@yahoo.com (D.D. Ganji).

0167-7322/$ – see front matter
http://dx.doi.org/10.1016/j.molliq.2013.11.005

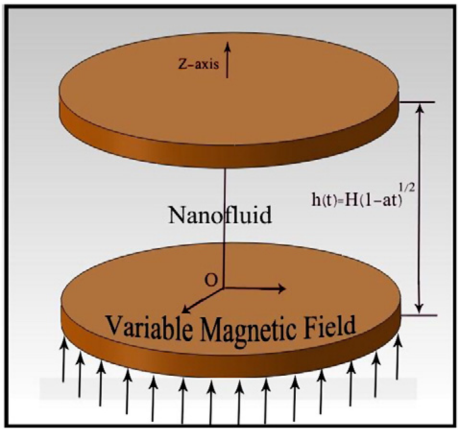

Fig. 1. Schematic of the problem, nanofluid between parallel plates in presence of variable magnetic field.

methods for solving the nonlinear heat transfer equations also Haji-Sheikh et al. [32,33] studied heat transfer in porous passage through weighted residual methods and series solution.

In the present study, the authors aim to investigate the heat transfer and fluid flow mechanism for nanofluid flow between parallel disks in presence of variable magnetic field. The innovative points of present study are introducing an exact and simple analytical method called LSM which does not needs to any linearization or perturbation, also the effect of constant parameters appeared in the mathematical section such as Hartmann number on velocity, temperature and nanoparticle concentration is investigated.

2. Problem description

Consider the incompressible two-dimensional flow of squeezing nanofluid between parallel disks separated by a distance $h(t) = H(1-at)^{1/2}$. A variable magnetic field of strength $B(t) = B_0(1-at)^{-1/2}$ is applied perpendicular to the disks. Fig. 1 shows the physical configuration of the problem. Here T_w and C_w denote the temperature and nanoparticle concentration at the lower disk while the temperature and concentration at the upper disk are T_h and C_h respectively. The upper disk at $z = h(t)$ moves towards or away from the stationary lower disk with the velocity dh/dt.

For $a > 0$, the two plates are squeezed until they touch $t = 1/a$ and for $a < 0$ the two plates are separated. The viscous dissipation effect, the generation of heat due to friction caused by shear in the flow, is retained. This effect is quite important in the case when the fluid is largely viscous or flowing at a high speed. The nanofluid is a two component mixture with the following assumptions: incompressible; no-chemical reaction; negligible viscous dissipation; negligible radiative heat transfer; nano-solid-particles and the base fluid are in thermal equilibrium and no slip occurs between them. The equations which governing the flow and mass transfer in viscous fluid are [12]:

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho_f}\frac{\partial p}{\partial r}+\upsilon\left(\frac{\partial^2 u}{\partial r^2}+\frac{\partial^2 u}{\partial z^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right), \tag{2}$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho_f}\frac{\partial p}{\partial z}+\upsilon\left(\frac{\partial^2 w}{\partial r^2}+\frac{\partial^2 w}{\partial z^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right), \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)+\tau\left[D_B\left(\frac{\partial C}{\partial r}\frac{\partial T}{\partial r}+\frac{\partial C}{\partial z}\frac{\partial T}{\partial z}\right)+\frac{D_T}{T_m}\left[\left(\frac{\partial T}{\partial r}\right)^2+\left(\frac{\partial T}{\partial z}\right)^2\right]\right] \tag{4}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial r}+w\frac{\partial C}{\partial z}=\left[D_B\left(\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}+\frac{\partial^2 C}{\partial z^2}\right)+\frac{D_T}{T_m}\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right]\right] \tag{5}$$

The boundary conditions are (see [11])

$$\begin{aligned} &z=h(t): \quad u=0, \quad w=\frac{dh}{dt}, \quad T=T_h, \quad C=C_h \\ &z=0: \quad u=0, \quad w=-\frac{w_0}{\sqrt{1-at}}, \quad T=T_w, \quad C=C_w \end{aligned} \tag{6}$$

where u and v are the velocity components in the r- and z-directions respectively and ρ is the density, μ is the dynamic viscosity, p is the pressure, T is the temperature, C is the nanoparticle concentration, α is the thermal diffusivity, D_B is the Brownian motion coefficient, D_T is the thermophoretic diffusion coefficient, T_m is the mean fluid temperature and k is the thermal conductivity. The last term in the energy equation is the total diffusion mass flux for nanoparticles which is given as a sum of two diffusion terms (Brownian motion and thermophoresis). Furthermore, τ is the dimensionless parameter which gives the ratio of effective heat capacity of the nanoparticle material to heat capacity of the fluid. So, the value of τ will be special for different fluids and nanoparticle materials. By using the following similarity transformations [4],

$$\begin{aligned} &u=\frac{ar}{2(1-at)}f'(\eta), w=-\frac{aH}{\sqrt{1-at}}f(\eta), \eta=\frac{z}{H\sqrt{1-at}} \\ &B(t)=\frac{B_0}{\sqrt{1-at}}, \theta=\frac{T-T_h}{T_w-T_h}, \phi=\frac{C-C_h}{C_w-C_h} \end{aligned} \tag{7}$$

By substituting these functions into Eqs. (2) and (3) and then eliminating the pressure gradient from the resulting equations and rewritten

Table 1
Sample values for LSM compared to NUM and % error for suction ($A = 1$) and upward motion of disk ($S = 1$) when $Nt = Nb = 0.5$, $M = 1$ and $Le = 2$.

η	$\theta(\eta)$			$f(\eta)$		
	NUM	LSM	% error LSM	NUM	LSM	% error LSM
0.0	1.000000000	1.0000000	0.000000	1.00000000000000	1.0000000000	0.00000
0.1	0.891588856045807	0.8920828085	0.000554	0.98295576366828	0.9828258529	0.00013
0.2	0.785972190997517	0.7871532425	0.001503	0.93943146178925	0.9389264155	0.00054
0.3	0.684473341420137	0.6855243591	0.001536	0.87888602681640	0.8780711978	0.00093
0.4	0.586826664512791	0.5870671295	0.00041	0.80883986653932	0.8080619937	0.00096
0.5	0.491925215300060	0.4912104383	0.00145	0.73552199174305	0.7351444347	0.00051
0.6	0.398247170606489	0.3969410843	0.00328	0.66435042686593	0.6645045189	0.000232
0.7	0.304067970657458	0.3028037802	0.00416	0.60030051476275	0.6008027130	0.000837
0.8	0.207556818950810	0.2069011522	0.00316	0.54820297463470	0.5486846850	0.000879
0.9	0.106828049003112	0.1068937408	0.000615	0.51300258558018	0.5131941787	0.000373
1.0	0.0000000	0.0000000	0.00000	0.50000000000000	0.5000000000	0.00000