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Forced convection analysis for MHD Al_2O_3–water nanofluid flow over a horizontal plate

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ABSTRACT

In this paper, forced-convection boundary-layer of MHD Al_2O_3–water nanofluid flow over a horizontal stretching flat plate is investigated using Homotopy Analysis Method (HAM) and fourth order Runge–Kutta numerical method. Comparison between HAM and numerical method shows that HAM is an exact and high efficient method for solving these kinds of problems. The influence of the nanofluid volume fraction (φ) and magnetic parameter (Mn) on non-dimensional temperature and velocity profiles is investigated. As an important outcome, by increasing Mn number, thermal boundary layer thickness significantly increased but increasing the nanofluid volume fraction hasn't very sensible effect on it.

1. Introduction

Most scientific problems in fluid mechanics and heat transfer problems are inherently nonlinear. All these problems and phenomena are modeled by ordinary or partial nonlinear differential equations. Most of these described physical and mechanical problems are with a system of coupled nonlinear differential equations. For an example heat transfer by natural convection which frequently occurs in many physical problems and engineering applications such as geothermal systems, heat exchangers, chemical catalytic reactors and forced convection of a nanofluid flow on a horizontal plate has a system of coupled nonlinear differential equations for temperature and stream function distribution equations which can be solved by analytical methods such as Homotopy Perturbation Method (HPM) [1].

Nanofluid, which is a mixture of nano-sized particles (nanoparticles) suspended in a base fluid, is used to enhance the rate of heat transfer via its higher thermal conductivity compared to the base fluid. Soleimani et al. [2] studied natural convection heat transfer in a semi-annulus enclosure filled with nanofluid using the Control Volume based Finite Element Method. They found that the angle of turn has an important effect on the streamlines, isotherms and maximum or minimum values of local Nusselt number. Natural convection of a non-Newtonian copper-water nanofluid between two infinite parallel vertical flat plates is investigated by Domairry et al. [3]. They conclude that as the nanoparticle volume fraction increases, the momentum boundary layer thickness increases, whereas the thermal boundary layer thickness decreases. Sheikholeslami et al. [4] performed a numerical analysis for natural convection heat transfer of Cu-water nanofluid in a cold outer circular enclosure containing a hot inner sinusoidal circular cylinder in presence of horizontal magnetic field using the Control Volume based Finite Element Method. Recently Sheikholeslami et al. [5] studied heat transfer of nanofluids in porous channels in the presence of magnetic field, also Aziz et al. [6] investigated the steady boundary layer free convection flow past a horizontal flat plate embedded in a porous medium filled by a water-based nanofluid containing gyrotactic microorganisms.

The term of Magnetohydrodynamic (MHD) was firstly introduced by Alfvén [7]. The theory of Magneto hydro dynamics is inducing current in a moving conductive fluid in the presence of magnetic field; such induced current results force on ions of the conductive fluid. The theoretical study of (MHD) channel has been a subject of great interest due to its extensive applications in designing cooling systems with liquid metals, MHD generators, accelerators, pumps and flow meters [8].

In recent years, much attention has been devoted to the newly developed methods to construct an analytic solution of equation; such methods include the Adomian decomposition method [9]; Perturbation techniques are too strongly dependent upon the so-called "small parameters" [10]. Thus, it is worthwhile developing some new analytic techniques independent upon small parameters. Homotopy Analysis Method (HAM), which was expected by Liao [11–13], has been applied to solve many types of nonlinear problems successfully [14–17].

The main motivation of this paper is to solve a two-dimensional forced-convection boundary-layer MHD problem in the presence of a magneto hydrodynamic (MHD) field over a horizontal flat plate including

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the viscous dissipation term using the HAM. The main advantage of this study is using an analytical method which does not need any small parameter and obtained results are compared with the numerical results which confirm the high accuracy of the applied method furthermore the effect of some parameters appeared in the mathematical formulation on stream function and temperature profile is investigated.

2. Description of the problem

Consider the two-dimensional forced convection boundary layer with variable magneto hydrodynamics (MHD) field of an incompressible flow including nanoparticles over a horizontal surface (see Fig. 1). By using boundary layer approximation and considering viscous dissipation term, the simplified two-dimensional equations governing the flow in the boundary layer of a steady, laminar, and incompressible flow are [1]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_{nf}}\left(\mu_{nf}\frac{\partial^2 u}{\partial y^2}-\sigma B(x)^2 u\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{nf}}{\left(\rho C_p\right)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where u and v are the x and y components of velocity respectively, σ is the electrical conductivity, $B(x)$ is the variable magnetic field acting in the perpendicular direction to the horizontal flat plate, μ_{nf} and ρ_{nf} are the viscosity and the density of the nanofluid respectively and α_{nf} is the thermal diffusivity and $(\rho C_p)_{nf}$ is the heat capacitance of the nanofluid. The appropriate physical boundary conditions are defined as below:

$$u=u_w=bx^m,\ \ v=0, T=T_w \tag{4}$$

$$u\to 0, T\to T_\infty \tag{5}$$

where, u_w is the x-component of velocity on the horizontal flat plate, b and m are constants, and T_w and T_∞ are the plate and ambient temperatures respectively. The nanofluid properties such as the density, ρ_{nf}, the dynamic viscosity, μ_{nf}, the heat capacitance, $(\rho C_p)_{nf}$, and the thermal conductivity, k_{nf} are defined in terms of fluid and nanoparticle properties as [18].

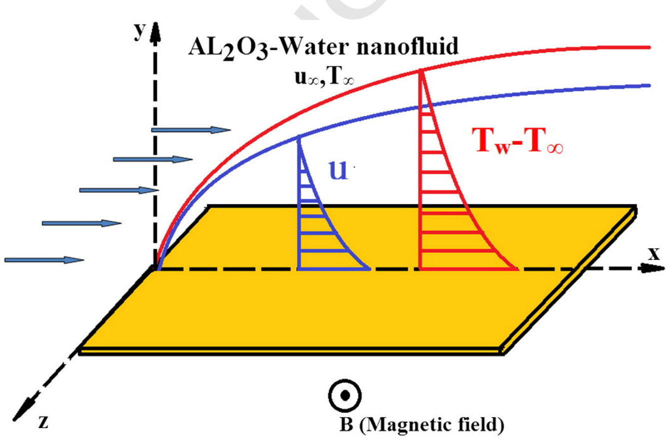

Fig. 1. Schematic of the nanofluid boundary layer on a horizontal plate in the presence of magnetic field.

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s \tag{6}$$

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}} \tag{7}$$

$$\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi\left(k_f-k_s\right)}{k_s+2k_f+2\phi\left(k_f-k_s\right)} \tag{8}$$

$$\left(\rho C_p\right)_{nf}=(1-\phi)\left(\rho C_p\right)_f+\phi\left(\rho C_p\right)_s \tag{9}$$

$$\alpha_{nf}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}} \tag{10}$$

where, ρ_f is the density of fluid, ρ_s is the density of nanoparticles, ϕ is defined as the volume fraction of the nanoparticles, μ_f is the dynamic viscosity of fluid, $(\rho C_p)_f$ is the thermal capacitance of fluid, $(\rho C_p)_s$ is the thermal capacitance of nanoparticles and k_f and k_s are the thermal conductivities of fluid and nanoparticles respectively. The variable magnetic field is defined as [19,20]:

$$B(x)=B_0\left(x^{\frac{m-1}{2}}\right) \tag{11}$$

where B_0 and m are constant. The following dimensionless similarity variable is used to transform the governing equations into the ordinary differential equations;

$$\eta=\frac{y}{x}{\mathrm{Re}_x}^{1/2} \tag{12}$$

$$\mathrm{Re}_x=\frac{\rho_f u_w(x)}{\mu_f}x. \tag{13}$$

The dimensionless stream function and dimensionless temperature are defined as:

$$f(\eta)=\frac{\psi(x,y)(\mathrm{Re}_x)^{1/2}}{u_w(x)} \tag{14}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{15}$$

where the stream function $\psi(x,y)$ is defined as:

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}. \tag{16}$$

By applying the similarity transformation parameters, the momentum equation (Eq. (2)) and the energy equation (Eq. (3)) can be rewritten as [1]:

$$f'''+\left((1-\phi)+\phi\left(\frac{\rho_s}{\rho_f}\right)\right)(1-\phi)^{2.5}\left(\frac{m+1}{2}\right)^2 ff'' -\left((1-\phi)+\phi\left(\frac{\rho_s}{\rho_f}\right)\right)(1-\phi)^{2.5}(m)f'^2-\left((1-\phi)^{2.5}\mathrm{Mn}\right)f'=0 \tag{17}$$

$$\theta''+\left((1-\phi)+\phi\frac{\left(\rho C_p\right)_s}{\left(\rho C_p\right)_f}\right)\mathrm{Pr}f\theta'+\frac{Ec\mathrm{Pr}}{(1-\phi)^{2.5}}f''^2=0. \tag{18}$$

Therefore, the transformed boundary conditions are:

$$f'(0)=1,\quad f(0)=0,\quad f'(\infty)=1 \\ \theta(0)=1,\quad \theta(\infty)=0. \tag{19}$$