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Bounds on fifth forces at the sub-Å length scale

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ABSTRACT

Constraints on a possible fifth-force interaction between hadrons are derived based on an analysis of results from laser precision frequency measurements of antiprotonic helium atoms, both $\bar{p}^4He^+$ and $\bar{p}^3He^+$ species, and from experiments on resonant formation rates of $dd\mu^+$-ions in muon-catalyzed fusion processes. A comparison is made between accurate experimental data and first-principles theoretical descriptions of the exotic systems within a quantum electrodynamical framework. The agreement between theory and experiment sets general limits on a possible additional hadron–hadron interaction, written in the form of a Yukawa potential $V_5(r) = \alpha_5 \exp(-r/\lambda)/r$, with λ representing the characteristic length scale associated with the mass of a hypothetical force-carrying particle via $\lambda = \hbar/(m_5 c)$. The laser spectroscopic data of antiprotonic helium set a constraint of $\alpha_5/\alpha_{EM} < 10^{-8}$ for $\lambda < 1$ Å, while the binding energy of the muonic molecular deuterium ion delivers a constraint of $\alpha_5/\alpha_{EM} \sim 10^{-5}$ for $\lambda < 0.05$ Å, where α_{EM} represents the strength of the electromagnetic interaction or the fine structure constant.

1. Introduction

Despite the great success of the Standard Model of physics in describing physical processes at the microscopic scale, it is not considered complete, for it does not encompass gravity. Furthermore, it lacks a description of Dark Matter and an explanation of the present accelerated expansion of the Universe, which may be associated with Dark Energy or repulsive gravitation. The concept of Dark Matter [1] can alternatively be explained in terms of a deviation from the law of gravity at large length scales, via various modified newtonian dynamics (MOND) theories [2]. String theory [3] predicts the existence of higher-order dimensions that may be compactified; this compactification is postulated to give rise to deviations from Newtonian gravity at short lengths scales varying from the sub-μm to the mm scale [4,5]. Recent results of laser spectroscopic measurements on muonic hydrogen ($\mu^- p^+$) [6,7] are in disagreement with similar studies in atomic hydrogen ($e^- p^+$) in particular for derived values for the proton size, at the level of 7σ. These deviations might be ascribed to deviations from quantum electrodynamics (QED), possibly a deviation from Coulomb's law of electromagnetism at short length scales. These examples illustrate the rationale to search for additional forces.

The present study focuses on phenomena in the QED-sector, for which the (Coulomb) interaction potential is given by:

$$V_{EM} = Z_1 Z_2 \frac{\alpha_{EM}}{r} \hbar c, \tag{1}$$

where the coupling strength is the fine structure constant $\alpha = \alpha_{EM} = e^2/4\pi\epsilon_0\hbar c$. Deviations from physical law could be expressed as a modification, or in the mathematically equivalent form, as an additional *fifth force*:

$$V_5 = N_1 N_2 \alpha_5 \frac{\exp(-r/\lambda)}{r} \hbar c = N_1 N_2 \alpha_5 Y(r) \hbar c, \tag{2}$$

where the prefactors N_1 and N_2 could relate to some *charge* under the fifth force. In the rest of the discussions we associate $N_{1,2}$ with the hadron numbers or atomic mass numbers of each nucleus, with the expectation that the hypothetical fifth-force *charge* is proportional to the particle number. The fifth force is parameterized by a generalized Yukawa potential for a certain effective range λ, which is associated with the mass of a hypothetical bosonic gauge particle of mass $m_5 = \hbar/\lambda c$, which would act as the force-carrying particle.

The dimensionless coupling constant α_5 may be related to the strength of any known interaction, e.g. electromagnetism or gravity. The latter is represented by:

$$V_G = N_1 N_2 \frac{\alpha_G}{r} \hbar c, \tag{3}$$

where α_G is the dimensionless coupling constant for gravity. If we take the proton mass as the mass scaling unit, then α_G can be related to gravitational constant G by the relation $\alpha_G = G m_p^2/(\hbar c)$. The ratio between the gravitational and electromagnetic coupling

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constants between two protons is $\alpha_G/\alpha_{EM} = 8.1 \times 10^{-37}$, assuming the inverse-square law behavior of the respective interactions holds.

Tests of the inverse-square law behavior of gravity have been carried out over an enormous distance scale from kilometers to submicrons, where the latter short-distance constraints are obtained from Casimir-force experiments [8]. Recently an analysis has been performed on a fifth-force contribution at the typical distance scale of chemical bonds, thus at length scales of 1 Å. Precision measurements on HD^+ ions [9,10] and H_2, D_2 and HD neutral molecules [11–14] in comparison with advanced QED calculations for the HD^+ ion [15,16] and for neutral hydrogen molecules [17,18] allowed for a determination of a constraint $\alpha_5/\alpha_{EM} < 10^{-9}$ at length scales of 1 Å and larger [19]. We note that analogous constraints for extra *lepton–hadron* interactions may be obtained from a comparison of very accurate experimental and theoretical results on simple atoms and ions [20], e.g. H, He, and He^+.

In the present study, these results are extended to shorter length scales by considering two exotic atomic or molecular systems. Recent results of laser spectroscopic experiments on antiprotonic helium [21] are interpreted in order to derive a constraint on α_5/α_{EM} in the interval 0.05–1 Å, which is possible due to the smaller separation between heavy particles in these exotic atoms. In addition, from the binding energy a weakly bound $(v = 1, K = 1)$ state in the $dd\mu^+$ system, determined by temperature-dependent formation rate measurements in muon catalyzed fusion [22], bounds of α_5/α_{EM} in the range 0.005–0.01 Å are derived. The spatial extent of the wavefunctions, actually $r^2\Psi^2(r)$, for some relevant states in the $dd\mu^+$ and $\bar{p}He^+$ systems are plotted in Fig. 1 to indicate the sub-Ångstrom length-scale accessed. Also drawn in Fig. 1 are the wavefunctions relevant to the tightest constraints obtained from the HD^+ system. An assumption is made for the present systems investigated here, similar to that in the analysis which provided constraints from molecules for $\lambda > 1$ Å, that the effects of gravitational, weak and strong interactions do not play a role. Thus, a comparison between experiment and theory can be made based on calculations solely in the domain of QED.

2. Antiprotonic helium

Antiprotonic helium ($\bar{p}He^+$) is an exotic neutral system composed of a helium nucleus with an antiproton replacing one of the two electrons in a He atom. This long-lived exotic atom, or molecule in view of the heavy interacting particles, was discovered some 20 years ago at the KEK accelerator facility in Japan [23]. Antiprotonic substitution takes place when antiprotons are brought to rest in a liquid helium target, where almost all antiprotons captured by the helium atom promptly annihilate in the subsequent encounter with the helium nucleus. A small fraction of the captured antiprotons, in particular, those in states occupying nearly circular orbitals around the He nucleus, is stable against collisions and may survive as long as several microseconds.

The surprising longevity allows for the manipulation of these $\bar{p}He^+$ states, e.g. by high precision measurements of laser induced transitions [24]. The accurate measurements of a set of one-photon transitions both in He-3 and He-4 isotopes [25] were included into the CODATA adjustment of the fundamental physical constants of 2006 and particularly of the (anti)proton-to-electron mass ratio. The fractional measurement accuracy of single-photon laser spectroscopy experiments of $\bar{p}He^+$, however, is limited by the Doppler effect.

More accurate results have been obtained recently from a Doppler-reduced two-photon laser spectroscopic experiment [21]. In order to enhance the two-photon transition probability, two counterpropagating laser beams of slightly unequal frequencies where used, with the frequency of one detuned by some 6 GHz from an intermediate state. Due to the near-equal frequencies of the counterpropagating beams the first-order Doppler effect largely cancels out, allowing for a more precise spectral line recording, where the hyperfine structure is partially resolved.

Accurate theoretical results for the three-body $\bar{p}He^+$ system where obtained in terms of power series expansion in the fine structure constant α. The nonrelativistic energies were obtained with an accuracy of 16 significant digits by using a variational expansion [26]. Since these states are truly resonant states, the Complex Coordinate Rotation (CCR) approach has been used in order to obtain square integrable wave functions, and the Rayleigh–Schrödinger perturbation theory is applied to an isolated CCR state [27]. Details of calculations may be found in [28]. Here we point out that the leading order relativistic corrections in the form of the Breit–Pauli Hamiltonian and the leading-order radiative correction considered account for recoil corrections, while the higher-order terms were taken within the nonrecoil approximation. The finite size corrections were also included, however, their contribution to the *vibrational* or *inter-Rydberg* transitions strongly cancels out, such that the uncertainty contribution of the uncertainty in the nuclear charge radii is negligible. The energy contributions for the $(n = 36, \ell = 34) \rightarrow (34, 32)$ transition in $\bar{p}^4He^+$ are listed in Table 1. The uncalculated higher-order QED terms as well as numerical errors in the calculations contribute to the total uncertainty in the theoretical transition energies.

The theoretical results and the most accurate experimental data from Hori et al. [21] on precision two-photon spectroscopy of the antiprotonic helium atoms, both in ^3He and ^4He, for the three observed transition are presented in Table 2. Recent progress in the calculation of the one-loop self-energy contribution of the

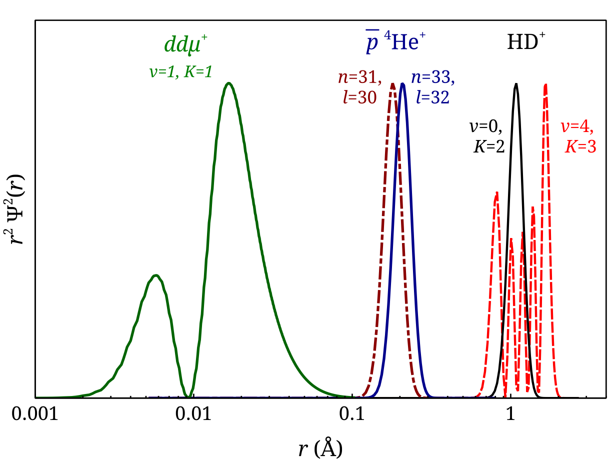

Fig. 1. The wavefunction densities of relevant states in the systems analyzed in the present study: the weakly bound (1,1) state of $dd\mu^+$ system, two states of antiprotonic-^{4}He involved in a measured two-photon transition, and two states involved in $R(2)$ transition the (4,0) band in the HD^+ ion. (v, K: vibrational, rotational quantum numbers).

Table 1
A list of contributions to the transitional frequency (in MHz) of the two-photon $(n = 36, \ell = 34) \rightarrow (34, 32)$ transition in the antiprotonic helium atom $\bar{p}^4He^+$. The uncertainty in the first parentheses is the contribution from higher-order terms, while that in the second is due to numerical errors.

ΔE_{nr} = 1522150208.13
ΔE_{α^2} = −50320.63
ΔE_{α^3} = 7069.5(0.3)
ΔE_{α^4} = 113.1
ΔE_{α^5} = −11.3(2.1)
ΔE_{total} = 1522107058.8(2.1)(0.3)

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