Microelectronics Journal 43 (2012) 904–907

Contents lists available at SciVerse ScienceDirect

Microelectronics Journal

journal homepage: www.elsevier.com/locate/mejo

Measurement of the time-constant spectrum: Systematic errors, correction

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ARTICLE INFO

Article history:
Received 30 November 2011
Received in revised form
4 May 2012
Accepted 23 May 2012
Available online 29 June 2012

Keywords:
Thermal characterization
Error correction
Systematic error
Time-constant spectrum

ABSTRACT

The transient thermal properties of an IC package are typically characterized by the thermal step-function response and/or by the time-constant spectrum. The temperature response is acquired from measurements or simulations while the time-constant spectrum is obtained from this response, using the NID method (Network Identification by deconvolution). The NID method is accurate only if the calculation is based on the exact step-function response. However, practical measurements provide us with responses which are more or less accurate but never absolutely exact. In our paper we present the sources of deviations and a method to eliminate their effect. We demonstrated the process on examples where the level of the corrected errors can be seen at various time-constants.

1. Introduction

In a thermal characterization problem the time-constant spectrum [1] is obtained from the thermal step-function, using the NID method [2]. The NID method requires an exact step-function response. In a general case the thermal step-function responses are provided by measurements that are never absolutely exact, we have to deal with deviations. There are other methods for thermal characterizations such as the multipoint moment matching method [3], however, in this paper we are dealing only with the NID method. The physical sources of the deviations are:

1. the onset of the step-function excitation does not occur exactly at the $t=0$ instant (per definition $t=0$ is the time instant corresponding to the 0 point of the time scale assigned to the measured response),
2. the rise time of the excitation is finite,
3. the cut-off frequency of the used measurement instrument is finite,
4. the measured object, the used temperature sensor and/or the measuring instrument suffers from slight nonlinearity.

It is highly needed to clarify the effect of these imperfections, in order:

- to have an image about the accuracy of the measurement and identification process currently used,
- to correct the systematic errors.

In this paper we are dealing with the first three sources of the deviations enumerated above. These effects can be handled on the basis of linear network theory. The nonlinear effects (fourth in the enumeration) have been discussed in an earlier paper from our research group [4].

This subject is relevant because e.g. an upcoming standard will describe the thermal transient measurement as a standard method to estimate the junction-to-case thermal resistance [5,6], therefore, the accuracy improvement is vital.

In the framework of our investigation the time-constant spectrum is regarded as the primary description function of the thermal one-port. We intended to treat the effect of the above imperfections as some characteristic distortion of the time-constant spectrum.

2. Time-constant spectrum

In this paragraph we summarize the definition of the time-constant spectrum, based on [7]. Owing to size limitations we do not present here the detailed discussions and proofs concerning these notions and relations. Despite of this, if the reader accepts the equations presented in this chapter, following the further parts of the paper must not raise difficulties.

A lumped element one-port can be represented by a finite number of τ time-constants and R magnitudes. A graphical representation of this is demonstrated in Fig. 1. Each line of this

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http://dx.doi.org/10.1016/j.mejo.2012.05.011

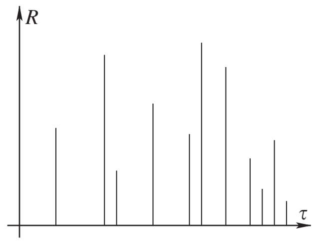

Fig. 1. Discrete time-constant distribution.

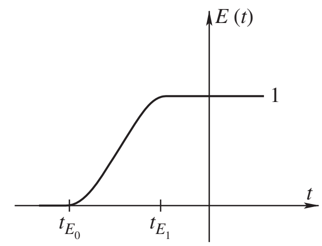

Fig. 2. Onset of the excitation.

plot corresponds to a time-constant and the height of the line is proportional to its magnitude. This figure can be regarded as some kind of a spectrum; the spectrum of time-constants appears in the step-function response of the network. Evidently the port-impedance of a lumped element network has discrete "spectrum lines" in finite number. An infinite distributed network has no discrete lines, but it is expected that some continuous spectrum would be suitable to describe them. The physical meaning of this idea is that in a general response any time-constant can occur in some amount, some density so that a density spectrum can be suitable to describe it.

Let us exactly define this spectrum function. First we introduce a new, logarithmic variable for time and time-constants. This choice leads to convolution-type equation for the network response, which offers an easy way for network identification (the NID method)

$$z = \log t \tag{1}$$

$$\zeta = \log \tau \tag{2}$$

Let us consider an RC one-port the response of which contains numerous exponentials having different time-constants and magnitudes. The *time-constant density* is defined as

$$R(\zeta) = \lim_{\Delta\zeta \to 0} \frac{\text{sum of magnitudes between } \zeta \text{ and } \zeta + \Delta\zeta}{\Delta\zeta} \tag{3}$$

Obviously this definition gives a density function on the logarithmic time-constant scale. From this definition it directly follows that the step-function response can be composed from the time-constant density

$$a(t) = \int_{-\infty}^{\infty} R(\zeta)\left[1 - \exp\left(-\frac{t}{\exp(\zeta)}\right)\right] d\zeta \tag{4}$$

In this interpretation an integral equation gives the time-constant density from the response function of the one-port. Derivation of this equation leads to a convolution type relation

$$\frac{da}{dz} = R(z) \otimes \exp(z - \exp(z)) \tag{5}$$

which is the base equation of NID identification method.

Time-constant spectrum can be defined on the linear time scale as well. Let be denoted this latter by $D(\tau)$. In the following calculations we always use this latter variant of the time-constant spectrum. This function shows one-to-one correspondence with $R(\zeta)$ spectrum defined by (3)

$$R(\zeta) = \exp(\zeta) \cdot D(\tau = \exp(\zeta)) \tag{6}$$

Based on this fact we will conclude that the further statements are valid for $R(\zeta)$ as well.

3. Effect of the non-ideal excitation

If the one-port is characterized by the $D(\tau)$ time-constant spectrum then its unit step response is written as

$$a(t) = \int_0^{\infty} D(\tau) \cdot (1 - \exp(-t/\tau))\, d\tau \tag{7}$$

if $t \geq 0$ else $a(t) = 0$.

The Dirac-δ response of the same one-port is

$$s(t) = \frac{da}{dt} = \int_0^{\infty} \frac{D(\tau)}{\tau} \exp(-t/\tau)\, d\tau \tag{8}$$

if $t \geq 0$ else $s(t) = 0$.

The non-ideal excitation is $E(t)$, which is 0 if $t < t_{E_0}$, 1 if $t > t_{E_1}$ (see Fig. 2). The derivative of this function is

$$e(t) = \frac{dE}{dt} \tag{9}$$

The $m(t)$ response that is measured using the actual excitation is calculated by the convolution integral

$$m(t) = e(t) \otimes s(t) = \int_{t_{E_0}}^{t_{E_1}} e(x) \cdot \underbrace{\int_0^{\infty} \frac{D(\tau)}{\tau} \exp(-(t-x)/\tau)\, d\tau}\, dx \tag{10}$$

The integral marked by the brace is zero if $t - x < 0$, that is if $t < x$ (see (8)). We intended to avoid this region during integration by x, assuming a condition of $t > t_{E_1}$. The region of the $m(t)$ measured function required by the time-constant identification is the $(0,\infty]$ time interval. This means that t_{E_1} should be less than zero in order to fulfill the condition. The excitation shown in Fig. 2 complies with this condition. The following derivation (Eqs. (11)–(15)) is correct if this condition is fulfilled

$$m(t) = \int_{t_{E_0}}^{t_{E_1}} \int_0^{\infty} e(x) \cdot \frac{D(\tau)}{\tau} \exp(-t/\tau) \exp(x/\tau)\, d\tau\, dx \tag{11}$$

We can regroup the parts of Eq. (11)

$$m(t) = \int_0^{\infty} \frac{D(\tau)}{\tau} \int_{t_{E_0}}^{t_{E_1}} e(x) \cdot \exp(x/\tau)\, dx \exp(-t/\tau)\, d\tau \tag{12}$$

This means that the D_m measured spectrum (in more correct phrasing: the spectrum belongs to the measured response and can be calculated from this response in ideal case referred here as D_m) can be written as

$$D_m(\tau) = D(\tau) \int_{t_{E_0}}^{t_{E_1}} e(x) \cdot \exp(x/\tau)\, dx = D(\tau) \cdot K(\tau) \tag{13}$$

where

$$K(\tau) = \int_{t_{E_0}}^{t_{E_1}} e(x) \cdot \exp(x/\tau)\, dx \tag{14}$$

$K(\tau)$ is a correction function which can be calculated if the rising function of the excitation is known. Possessing this function the correction of the time-constant spectrum can be performed by