Surface Science 641 (2015) 91–96

Contents lists available at ScienceDirect

Surface Science

journal homepage: www.elsevier.com/locate/susc

Topological Mott insulator by block spin phenomenology

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ARTICLE INFO

Article history:
Received 19 November 2014
Accepted 20 May 2015
Available online 27 May 2015

Keywords:
Topological insulator
Block spin
Spin glass

ABSTRACT

We investigate the relationship between topological Mott insulators and spin glasses.
By first explaining the phase of spin glass on the basis of finite sized block spin concepts, we then introduce the three-dimensional insulating phase of a topological insulator with a finite bulk bandgap as the pairing of block spins comprised of many random spins with respective majority spin directions. However, the two-dimensional edge state of the topological insulator may be thought of as the pairing of triplet spins with a zero bandgap. Topological insulators can be transformed into ordinary insulators below a certain temperature. Electric field-induced transitions between normal and topological insulators are possible as explained by means of composite charges.

1. Introduction

The common characteristics of topological insulators (TIs) [1] include two-dimensional (2D) conducting states on their edge or surface and a three-dimensional (3D) insulating phase with a bulk bandgap [2]. The concept of a TI was first postulated for graphenes, in which a 3D insulator [3] can be compatible with 2D conducting edge states as a result of strong spin-orbit effects as described by Kane and Mele [4]. This means that most powerful property of a TI is related to the band crossing from numerical band calculations, leading to conducting surfaces. The major recent theoretical approaches to TIs are based on topological field theories [5] and mean field schemes [6]. Experimental observations of HgTe/CdTe quantum well structures [7] and $Bi_{1-x}Sb_x$ [8], among others, show the 2D quantum spin Hall effects and gapped states in bulk. Topological Mott insulators and topological Anderson insulators categorised as TIs have been the subject of considerable interest for research groups, and the problem of the coexistence of superconductors or quantum Hall conductors in 2D and 3D insulators has also attracted the attention of both theorists and experimentalists.

In this paper, we employ renormalisation group theory [9] and its compatibility with finite-sized block spin concepts, in which each spin block is of such a size as to yield a finite nonzero total spin, meaning that we can assign the normal states of TIs as spin glasses.

2. Theory

We postulate spin glasses to be composed of spin clusters that may be treated as block spins. [10] In the presence of an external magnetic field H in the z-direction, the Hamiltonian for a spin glass comprised of random block spins is given by [11]

$$\begin{aligned} H &= \sum_{i=1} g\mu_B \vec{H}\cdot\vec{S}_i \\ &= g\mu_B H \sum_{i=1} S_{iz} \end{aligned} \tag{1}$$

where g is the Lande's factor for a block spin, μ_B is the Bohr magneton, and $\vec{S}_i$ is the spin operator for a block spin.

The block spin magnetisation is then given by

$$\begin{aligned} \langle M_z\rangle &= -g\mu_B \sum_i \langle S_{iz}\rangle \\ &= -N_B g\mu_B \langle S_z\rangle \end{aligned} \tag{2}$$

$$\langle M_z\rangle = -N_B g\mu_B \left\{ \sum_{S_z=-S}^{S} S_z \exp[-g\mu_B H S_z/(k_B T)] \right\} \Big/ Z$$

$$Z = \sum_{S_z=-S}^{S} \exp[-g\mu_B H S_z/(k_B T)]$$

$$S = \delta\left(\frac{1}{2}\right)N$$

where N_B is the number of block spins, N is the number of random spins in a block spin, and $\delta \approx 0$ represents an infinitesimal deviation from a zero average.

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http://dx.doi.org/10.1016/j.susc.2015.05.019
0039-6028/

The resultant magnetisation and the freezing temperature T_f are then obtained from the mean field average for the annealed random system as

$$\begin{aligned}
&<M_z> = N_B g\mu_B S B_S[g\mu_B HS/(k_B T)]\\
&B_S[x] = \frac{2S+1}{2S}\coth\left(\frac{2S+1}{2S}x\right) - \frac{1}{2S}\coth\left(\frac{1}{2S}x\right)\\
&B_S(x \approx 0) \cong \frac{S+1}{3S}x\\
&B_S(x >> 1) \simeq 1 - \frac{1}{S}e^{-\frac{x}{S}}\\
&\frac{S+1}{3S}g\mu_B HS/(k_B T_f) = 1 - \frac{1}{S}e^{-\frac{g\mu_B HS/\left(k_B T_f\right)}{S}} + a_0 = \frac{1}{S} - \frac{1}{S}e^{-\frac{g\mu_B HS/\left(k_B T_f\right)}{S}}\\
&<M_z>|_{T=\infty} = 0
\end{aligned} \tag{3}$$

where the Brillouin function is approximated in the two asymptotic limits and a_0 is a constant for compensation.

The real and imaginary parts of the susceptibility are then calculated respectively as

$$\begin{aligned}
&\chi' = \frac{\partial}{\partial H}<\mathrm{Re}(M_z)>\\
&\chi'' = \frac{\partial}{\partial H}<\mathrm{Im}(M_z)>\\
&\chi = \chi' + i\chi''\\
&\mathrm{Im}<M_z> = N_B g\mu_B S\tilde{B}_S[g\mu_B HS/(k_B T)]\\
&\tilde{B}_S[x] = \frac{2S+1}{2S}\tanh\left(\frac{2S+1}{2S}x\right) - \frac{1}{2S}\tanh\left(\frac{1}{2S}x\right)
\end{aligned} \tag{4}$$

where the imaginary part corresponds to $f\left(\frac{\pi}{2}-\theta\right)$ for the real part of $f(\theta)$ with the phase angle variable θ chosen arbitrarily.

Following Curie-Weiss theory [11], the ferromagnetic transition temperature is given by

$$T_{FM} = \frac{2z}{3k_B}S(S+1)J \tag{5}$$

with the magnetic susceptibility given by

$$\begin{aligned}
&\chi_H = \frac{C}{T - T_{FM}}\\
&C = N_B(g\mu_B)^2 S(S+1)/3k_B
\end{aligned} \tag{6}$$

where k_B is Boltzmann's constant, z is the number of nearest neighbour block spins, and J is the exchange integral between the nearest block spins in the renormalisation group transformations.

The average number density of an electron in the absence or presence of H is given by

$$\begin{aligned}
&\int_0^\infty f(\varepsilon)d\varepsilon = \int_0^\infty \frac{d\varepsilon}{1+\exp\frac{\varepsilon-\varepsilon_F}{k_B T}} = \int_0^\infty \frac{d\varepsilon}{1+\exp\frac{\varepsilon \pm \mu_B H \pm eEL \pm \hbar\omega - \varepsilon_F}{k_B T_{eff}}}\\
&\equiv k_B T \ln\left[1+\exp\frac{\varepsilon_F}{k_B T}\right] = k_B T_{eff}\ln\left[1+\exp\frac{-(\pm\mu_B H \pm eEL \pm \hbar\omega - \varepsilon_F)}{k_B T_{eff}}\right]
\end{aligned} \tag{7}$$

where $f(\varepsilon)$ is the Fermi-Dirac distribution, $N(\varepsilon)$ is the density of states, β_i are positive constant parameters, and ε_F is the Fermi energy.

Here the effective temperature is given by

$$k_B T_{eff} \equiv k_B T \pm \beta_H \mu_B H \pm \beta_E eEL \pm \beta_\omega \hbar\omega.$$

Let us now consider the distribution of anyonic block spins.

The number density is then given as

$$\begin{aligned}
&<\tilde{n}_k> = \frac{tr\left(e^{-\beta\sum_l(\varepsilon_l-\mu)\tilde{n}_l}\tilde{n}_k\right)}{Z}\\
&\tilde{n}_k = S, S-1, S-2, \ldots\ldots, 0\\
&Z = tr\left(e^{-\beta\sum_l(\varepsilon_l-\mu)\tilde{n}_l}\right)
\end{aligned} \tag{8}$$

and using derived relationships [11] such as (9)

$$<\tilde{n}_k> \Leftrightarrow \frac{\frac{\partial}{\partial\{\beta(\varepsilon_k-\mu)\}}tr\left(e^{-\beta\sum_l(\varepsilon_l-\mu)\tilde{n}_l}\right)}{Z}$$

the distribution function of anyonic block spins can be rewritten as

$$\begin{aligned}
&f_{anyon} = \frac{S}{2}B_{\frac{S}{2}}\left(g\mu_B H\frac{S}{2}\beta\right) + \frac{S}{2}\\
&\text{Substituting } g\mu_B H \Rightarrow g\mu_B H \pm \frac{\beta_E}{\beta_H}eEL \pm \hbar\omega \pm \sqrt{\varepsilon^2+\Delta^2}\\
&\beta = \frac{1}{k_B T}
\end{aligned} \tag{10}$$

where ε, ε_l represent kinetic energies, and μ is the chemical potential.

Using the BCS scheme [12], the superconducting gap Δ and bandgap E_g from the triplet-like pairing of block spins with parallel spin configuration, $\left(\frac{S}{2},\frac{S}{2}\right)$ can be obtained as

$$\begin{aligned}
&-1 = N(\varepsilon_F)(|U_{BCS}+U_c|)\int_0^{\hbar\omega}\frac{d\varepsilon\left[SB_{\frac{S}{2}}\left\{gS\sqrt{\varepsilon^2+\Delta^2}/(2k_B T)\right\}+S-1\right]}{\sqrt{\varepsilon^2+\Delta^2}}\\
&\approx N(\varepsilon_F)(|U_{BCS}+U_c|)\left[\frac{S\frac{S}{2}\left(\frac{S}{2}+1\right)}{3\frac{S}{2}S}\frac{g\hbar\omega}{2k_B T} + (S-1)\int_0^{\hbar\omega}\frac{d\varepsilon}{\sqrt{\varepsilon^2+\Delta^2}}\right]\\
&-1 = N(\varepsilon_F)(|U_{BCS}+U_c|)\int_0^{\hbar\omega}\frac{d\varepsilon\left[S\tilde{B}_{\frac{S}{2}}\left\{gS\sqrt{\varepsilon^2+{E_g}^2}/(2k_B T)\right\}+S-1\right]}{\sqrt{\varepsilon^2+{E_g}^2}}\\
&\approx N(\varepsilon_F)(|U_{BCS}+U_c|)\left[-\frac{S\frac{S}{2}\left(\frac{S}{2}+1\right)}{3\frac{S}{2}S}\frac{g\hbar\omega}{2k_B T} + (S-1)\int_0^{\hbar\omega}\frac{d\varepsilon}{\sqrt{\varepsilon^2+{E_g}^2}}\right]\\
&E_g = \frac{\hbar\omega}{\sinh\left[\frac{1}{1-S}\left\{\frac{1}{N(\varepsilon_F)(|U_{BCS}+U_c|)} - \frac{S\frac{S}{2}\left(\frac{S}{2}+1\right)}{3\frac{S}{2}S}\frac{g\hbar\omega}{2k_B T}\right\}\right]}
\end{aligned} \tag{11}$$

where the real value of $\Delta_k \to$ imaginary value of E_g, and E_g is the band gap while the imaginary part is $f\left(\frac{\pi}{2}-\theta\right)$ for the real part of $f(\theta)$ with the phase angle θ, and $N(\varepsilon_F)$ is the density of states at the Fermi level. If $V = U_{BCS} + U_c > 0$, the superconducting gap becomes zero but the band gap is nonzero. U_{BCS} (U_c) is the BCS-type phonon-mediated interaction (Coulomb interaction) between block spins, which are obtained in the effective single electron approximation of block spins with scaled spin values, with appropriate values of effective mass and effective charge, $m^* = Nm_e >> m_e$ (m_e: the mass of bare electron), $e^* = Ne$, respectively. Here, if the triplet pairs occur two-dimensionally, i.e., $1 = S$, then