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An analytical model for calculating the pull-in voltage of micro cantilever beams subjected to tilted and curled effects

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ABSTRACT

Pull-in is a fundamental phenomenon in electrostatic micro devices. In previous studies on modeling the pull-in voltage of suspended micro cantilever beam subject to residual stress, only curled deformation was considered. This study proposed a modified deformation function, which considered both curled and tilted deformations caused by gradient stress and mean stress, to calculate the pull-in voltage of the suspended cantilever beam with residual deformations.

In order to verify the proposed analytical model, suspended poly-silicon cantilever beams with three different lengths, 260 μm, 295 μm and 330 μm, are fabricated through surface micromachining process. It is shown that the residual deformations include both curled and tilted deformations, where the tilted angle and radius of curvature can be identified by white light interferometer (WLI). By comparing the analytical results with measurement results on pull-in voltages, it is found that while only considering curled effect, the average error of calculated pull-in voltage is 10.5%. On the other hand, when both tilted and curled effects are considered, the average error is reduced to 3.2%, which verifies the accuracy improvement of the proposed analytical model.

1. Introduction

Electrostatically actuated micro cantilever beams are widely used in Micro Electro Mechanical Systems (MEMS), such as microsensors [1,2], microactuators [3,4], and AFM structures [5]. For electrostatic transducers, pull-in is a basic phenomenon, and its instability is fundamental to the understanding of many MEMS devices [1,3]. The pull-in voltage can be used to determine the material properties, such as Young's modulus, plate modulus, and residual stress [6]. Accurate model on the pull-in voltage is helpful in designing electrostatic MEMS devices.

Most of previous works on pull-in voltage modeling were limited to straight shapes or forms free from residual stress [7–9]. However, residual deformations are very common in suspended micro structures, especially fabrication by surface micromachining process. Therefore, models to calculate pull-in voltage of cantilever beam subjected residual stress were also reported [10–13] by considering the curled deformation of the cantilever beams due to the gradient stress. Wei et al. [10] modified straight beam model to include curvature of beam. Hu investigated pull-in voltage calculation for curled cantilever beams by Euler–Bernoulli beam theory, Taylor's series expansion, and energy method [11]. The analytical models were further modified by considering fringing field effect [12] and the elastic boundary effect of the anchor point [13]. However, residual stress may include both gradient stress and mean stress. In that case, cantilever beam will deflect out-of-plane, with its far field curvature being generated exclusively by gradient stress and with an initial slope determined by both gradient stress and mean stress [14]. It means that the residual deformations of a suspended cantilever subject to both gradient stress and mean stress may have both curled and tilted deformations in general. Therefore, the corresponding pull-in voltage calculation should also consider both curled and tilted effects.

The purpose of this research aims to improve the pull-in voltage calculation by including both tilted and curled effects of suspended cantilevers in a modified deformation function. Cantilevers made of poly-silicon with different lengths at different locations will be fabricated and tested to examine the accuracy of the proposed model.

2. Model

For a flat micro cantilever beam with length L, width b, thickness h, and a Young modulus E, subjected to a downward electrostatic load, the pull-in voltage V_{PI} can be expressed as [9]:

$$V_{PI} = \sqrt{\frac{2h^3 G_0 E/(1-\upsilon^2)}{8.37\varepsilon L^4\left(\frac{5}{6G_0^2} + \frac{0.19}{G_0^{1.25}b^{0.75}} + \frac{0.19}{G_0^2 L^{0.75}} + \frac{0.4h^{0.5}}{G_0^{1.5}b}\right)}} \quad (1)$$

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where G_0, ε, and ν are the initial air gap, permittivity of air, and Poisson's ratio, respectively. In Eq. (1), the beam is assumed to be initially straight with no axial tension/compression. However, the cantilever beam may have out-of-plane deformation after releasing the sacrificial layer in surface micromachining process, as shown in Fig. 1, therefore the predicted pull-in voltage by Eq. (2) could deviate significantly from the actual value. Since there is no constraint on the cantilever's free edge, the gradient component on the other hand provides a sensibly constant bending moment. As a result of such loading, the beam will curl as sketched in Fig. 1(b). Also, the boundary condition for anchor could involve zero displacement but a specified slope, as shown in Fig. 1(c), which is induced by the mean and gradient stresses in the original beam [14]. In short, under a general residual stress with gradient stress and mean stress, the cantilever beam may have out-of-plane curled and tilted deformations, with its far field curvature being generated exclusively by gradient stress and with an initial slope determined by both gradient and mean stress, as shown in Fig. 1(d).

To develop the analytical pull-in voltage solution of a cantilever beam with deformation, a mathematical function is needed to describe the deformation. For example, the curled deformation can be expressed as a function of the position x [10–13]:

$$G(x) = g_0 + \rho\left(1 - \cos\frac{x}{\rho}\right) \tag{2}$$

where g_0 is the initial gap between the fixed end of the cantilever beam and the ground plane, and ρ is the radius curvature of the deformed beam. Here, a modified deformation function including both tilted and curled deformations on cantilever beam is proposed:

$$G(x) = g_0 + \rho\left(1 - \cos\frac{x}{\rho}\right) + \theta \cdot x \tag{3}$$

where θ represents the tilted angle of the beam caused by gradient and mean stress.

The analytical pull-in voltage solution can be derived by energy method [15]. For a deformed cantilever beam subjected to a uniform electrical field, the total potential energy is the sum of the bending strain energy (U_m, based on the assumption of the Euler–Bernoulli beam) and the electrical potential energy U_e, i.e.

$$U = U_m + U_e = \frac{EI}{2}\int_0^L \left(\frac{d^2\omega}{dx^2}\right)^2 dx - \int_0^L \frac{\varepsilon b V^2}{2(G-\omega)}dx \tag{4}$$

where E, I, L, V, G, ω, ε, b, represent Young's modulus, the cross-sectional area moment of inertia, the beam length, the applied bias voltage, the initial gap between the beam and ground plane at position x, the deflection function at position x, the permittivity of air, and the beam width, respectively. Then, the deflection function $\omega(x)$ is assumed as:

$$\omega(x) = \eta\varphi(x) \tag{5}$$

where $\varphi(x)$ is the assumed deflection shape function satisfying the boundary conditions, and the coefficient η to be solved is the associated modal participation factor. At the transition from a stable to an unstable equilibrium state, the first-order and second-order derivatives of the total potential energy with respect to η both equal to zero, i.e.

$$\frac{\partial U}{\partial \eta} = EI\,\eta\int_0^L (\varphi'')^2 dx - \varepsilon b V^2\int_0^L \frac{\varphi}{2(G-\eta\varphi)^2}dx = 0 \tag{6}$$

$$\frac{\partial^2 U}{\partial \eta^2} = EI\int_0^L (\varphi'')^2 dx - \varepsilon b V^2\int_0^L \frac{\varphi^2}{(G-\eta\varphi)^3}dx = 0 \tag{7}$$

Solving Eq. (6) can determine η_{PI}, and then solving Eq. (7) can lead to the closed form of the pull-in voltage V_{PI} as:

$$V_{PI} = \sqrt{\frac{EI}{\varepsilon b} \times \frac{\int_0^L (\varphi'')^2 dx}{\int_0^L \frac{\varphi^2}{G^3}dx + 6\eta_{PI}\int_0^L \frac{\varphi^3}{G^4}dx + 12\eta_{PI}^2\int_0^L \frac{\varphi^4}{G^5}dx}} \tag{8}$$

By substituting the proposed modified deformation function $G(x)$, Eq. (3), into Eq. (8), the corresponding pull-in voltage solution considering both curled and tilted effects can be found. Similarly, substituting Eq. (2) into Eq. (8) can lead to the analytical pull-in voltage solution considering only curled effect for comparison. The integral terms in Eq. (8) are all constants related to the geometrical parameters of the beam, including ρ and θ in $G(x)$, which need to be identified experimentally. (The detailed solutions of $\varphi(x)$ and η_{PI} are listed in Supplementary data).

3. Experiments

3.1. Fabrication of cantilever beams

To examine the accuracy of the proposed model, cantilevers made of poly-silicon with different lengths at different locations are fabricated. The geometrical parameters and material properties of cantilever beams are listed in Table 1. Those beams are named as CB1, CB2, and CB3 for length 260, 295 and 330 μm, respectively.

The fabrication process of poly-silicon beams is illustrated in Fig. 2: (a) depositing gate oxide for CMOS circuit, (b) depositing the first poly-silicon layer and defining the wire and electrode, (c) depositing the isolation layer to avoid the electrical breakdown when pull-in happens, (d) depositing silicon oxide as sacrificial layer and defining the anchor of cantilever beam, (e) depositing second poly-silicon layer as structure layer, (f) releasing sacrificial layer to form suspended cantilever beam.

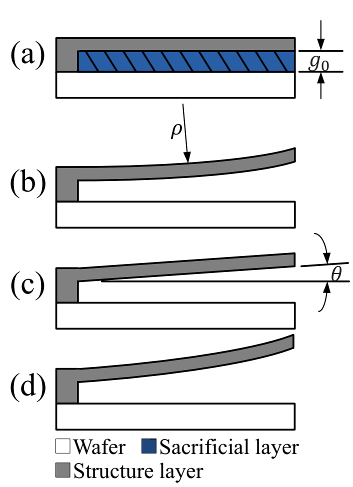

Fig. 1. States of a cantilever beam. (a) Initial state; (b) curled deformation; (c) tilted deformation; (d) deformation with both curled and tilted effects.

Table 1
The geometrical and material parameters of the cantilever beams.

Parameters	Descriptions	Values
L	Beam length	260/295/330 μm
b	Beam width	20 μm
h	Beam thickness	4.8 μm
g_0	Initial gap at fixed end	2 μm
L_a	Anchor length	2 μm
b_a	Anchor width	18 μm
h_a	Anchor thickness	4 μm
E	Young's modulus	160 GPa
ε	Permittivity of air	8.85×10^{-12} F/m