Journal of Quantitative Spectroscopy & Radiative Transfer 202 (2017) 31–36

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Angular scattering of an Airy beam light sheet by a concentric sphere

Zhaolou Cao [a,b,*], Chunjie Zhai [c]

[a] Jiangsu Key Laboratory for Optoelectronic Detection of Atmosphere and Ocean, Nanjing University of Information Science and Technology, Nanjing 210044, China
[b] School of Physics and Optoelectronic Engineering, Nanjing University of Information Science and Technology, Nanjing 210044, China
[c] Department of Forest Fire Protection, Nanjing Forest Police College, Nanjing 210046, China

ARTICLE INFO

Article history:
Received 1 May 2017
Revised 18 July 2017
Accepted 18 July 2017
Available online 20 July 2017

Keywords:
Airy beam
Concentric sphere
Angular scattering
Mie theory

ABSTRACT

Angular distribution of scattered light by a particle reveals information of its topology and refractive index, which has been proved to be an effective method to identify and characterize the particle. In this paper, numerical study on angular scattering of a polarized one-dimensional Airy beam light sheet by concentric spheres is presented. Plane-wave spectrum method is adopted to simplify the problem into classic Mie scattering problem, which significantly improves computational efficiency. The effects of size parameters, refractive index and position of concentric spheres and Airy beam radius on the angular distribution of scattered light are investigated. This paper provides theoretical results for light scattering measurement where Airy beam light sheet is employed for reducing effects of multiple scattering.

1. Introduction

Angular scattering pattern generated by a particle in a light field reveals the particle's properties, such as its shape and structure. As the pattern is extremely sensitive, light scattering technique was proposed to measure particles with sub-wavelength size. Since the technique is real-time, non-invasive and label-free, it has attracted much attention in many scientific fields in the last decades. Many instruments for different applications have been developed based on this technique. In these applications, angular distribution of scattered light is measured and interpreted by a numerical model to retrieve the scatterer information for further processing. Typical applications include flow cytometry [1,2] and polymer molecular weight measurement [3,4]. The former can classify different types of cells while the latter has been proven to be a promising method to obtain polymer's molecular weight. Conventionally, plane wave is employed in these measurements, where scattering is independent on the particle position. But in an active imaging system, narrow Gaussian beam that confines the energy in a small volume is usually used for its high spatial resolution. Unlike plane wave, Gaussian beam scattering is significantly affected by the position of particles. Theoretical and experimental works on Gaussian beam scattering have already been conducted by many researchers [5–7]. One-dimensional Gaussian beam was recently introduced in the light scattering measurement [8–9] of an imaging flow cytometry for high throughput and suppression of background noise from other scatterers. Promising results in phytoplankton analysis and label-free characterization of senescent cells demonstrate the great potential of this technique. However, both plane wave scattering and Gaussian beam scattering suffer from multiple scattering, which introduces unpredictable noise and system errors that can hardly be removed in postprocessing.

Nondiffractive beam, which can reconstruct its wavefront after being scattered by particles, is a natural choice to reduce the effects of multiple scattering. Bessel beam and Airy beam are two typical nondiffractive beams. Theoretical works on Bessel beam scattering have already been reported in many literatures. The scatterers include homogeneous dielectric sphere [10], concentric spheres [11] and uniaxial anisotropic biospheres [12]. While Bessel beam is always two-dimensional, Airy beam can be either one-dimensional or two-dimensional. One-dimensional Airy beam light sheet can then be employed to replace the one-dimensional Gaussian beam in Ref. [8,9]. While Airy beam has already found many applications, such as light sheet microscopy [13], optical manipulation [14] and imaging system with extended depth of field [15], Airy beam scattering hasn't got enough attention yet. In Ref. [16], we presented the study on scattering by a homogenous sphere in a one-dimensional light sheet. But particles in nature or artificially produced, such as biological cells, bacteria, coated particles and aerosols, are often found to be inhomogeneous, which can usually be approximated to be spheres with concentric spherical inclusion. Therefore, we focus on the angular scattering of one-dimensional Airy beam light sheet by concentric spheres in this paper, aim-

* Corresponding author at: School of Physics and Optoelectronic Engineering, Nanjing University of Information Science and Technology, Nanjing 210044, China.
E-mail address: zhaolou.cao@gmail.com (Z. Cao).

http://dx.doi.org/10.1016/j.jqsrt.2017.07.019

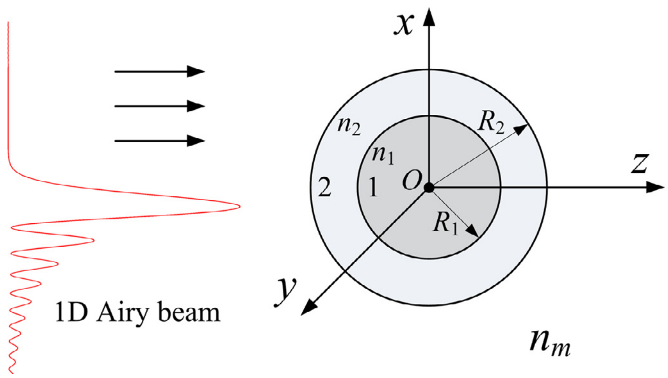

Fig. 1. Illustration of scattering of Airy beam light sheet by a concentric sphere consisting of core and shell. The coordinate system is used throughout the following text.

ing at providing a theoretical model for the beam's application in imaging flow cytometry.

The rest of this paper is organized as follows. Section 2 gives the description of the problem and theoretical treatment of the scattering problem. Section 3 presents the simulation results of the proposed model. Conclusions are drawn in Section 4.

2. Theory and description of the problem

Aden [17] first gave the analytical solution of scattering of concentric spheres in plane wave in the spherical coordinate system utilizing Mie theory. As the solution consists of infinite series, Toon [18] developed a numerically stable algorithm for the calculation of light-scattering intensity functions. Beam shape coefficients (BSC) are then employed to extend these works for scattering of shaped beam by concentric spheres, which have already been applied in Gaussian beam scattering and Bessel beam scattering. However, the calculation of BSC is usually time-consuming as it needs three-dimensional integration of radial component of the beam over a closed surface. Although it can be accelerated by local approximation, additional numerical error may be introduced. For practical purpose, high computational efficiency and accuracy will certainly be desirable. Therefore, plane-wave spectrum method is adopted in this paper, where one-dimensional Airy beam is expressed as superposition of plane waves. Far-field scattered light field of Airy beam can then be regarded to be superposition of scattered light field of each plane wave. Since Ref. [18] has already given the analytical solution of scattering of plane wave by concentric spheres, only one-dimensional integration is needed to get the far-field intensity of Airy beam scattering.

2.1. Expression of one-dimensional Airy beam

Scattering of Airy beam light sheet by a concentric sphere is schematically illustrated in Fig. 1. The beam propagates along the z-direction with polarization of x-direction in an infinite, isotropic and homogeneous medium. The coordinate system $Oxyz$ is attached to the sphere whose center is located at O. The corresponding spherical coordinates are then designated as (r, θ, φ).

As monochrome light is studied in this paper, the time-harmonic factor of exp($-i\omega t$) associated with the fields is omitted in this work. Vector potential, given by $\vec{A} = A(x,z)\vec{x}$, is employed to derive the analytical expression of Airy beam, where $\vec{x}$ is unit vector in the x-direction.

$$A(x,z) \propto \int_{-1}^{1} dn_x F(n_x) \exp\left\{ i f^i(x,z) \right\}, \tag{1}$$

$$f^i(x,z) = n_m k_0 [n_x(x - x_0) + n_z(z - z_0)], \tag{2}$$

where n_m is the refractive index of the medium, $k_0 = 2\pi/\lambda$ is wave vector, λ is the wavelength in the vacuum, (n_x, n_z) are the direction cosines of the wave vector with $n_x \in (-1, 1)$ and $n_z \in (0, 1]$, (x_0, z_0) denotes the center position of the beam,

$$F(n_x) = \frac{1}{2} \exp\left(-a_0 x_r^2 k_m^2 n_x^2\right) \exp\left[\frac{i}{3}\left(x_r^3 k_m^3 n_x^3 - 3a_0^2 x_r k_m n_x - i a_0^3\right)\right], \tag{3}$$

where $k_m = n_m k_0$, x_r is the transverse scale that determines the width of the beam's main lobe, a_0 is a positive parameter to ensure that the beam has finite energy and can be physically realized. With vector potential, electromagnetic field can be expressed as

$$\vec{E}^i = \frac{i}{n_m^2 k_0} \nabla \times \nabla \times \vec{A}, \tag{4}$$

$$\vec{H}^i = \nabla \times \vec{A}, \tag{5}$$

where subscript i indicates the incident light beam. Six components of the electromagnetic field are then obtained by substituting Eq. (1) into Eqs. (4) and (5), given by

$$E_x^i = E_0 \int_{-1}^{1} n_z^2 F(n_x) \exp\left[i f^i(x,z)\right] dn_x, \tag{6}$$

$$E_z^i = -E_0 \int_{-1}^{1} n_x n_z F(n_x) \exp\left[i f^i(x,z)\right] dn_x, \tag{7}$$

$$H_y^i = n_m E_0 \int_{-1}^{1} n_z F(n_x) \exp\left[i f^i(x,z)\right] dn_x, \tag{8}$$

$$E_y^i = H_x^i = H_z^i = 0, \tag{9}$$

where E_0 is a constant parameter defining the energy of the beam.

2.2. Airy beam scattering

As indicated by Eqs. (6)–(9), one-dimensional plane-polarized Airy beams are linear integration of plane waves that propagates in the xz plane. Therefore, scattering light field of Airy beam is also linear superposition of that of plane waves. We followed the work in Ref. [18] to numerically solve the plane wave scattering. Assuming the plane wave is x-polarized and $n_m = 1$, the far-field scattered light field is given by

$$\begin{aligned} E_\theta^s &= \frac{i \exp(ikr)}{kr} \cos\varphi S_2(\cos\theta) \\ E_\varphi^s &= \frac{\exp(ikr)}{ikr} \sin\varphi S_1(\cos\theta) \end{aligned} \tag{10}$$

where E_θ and E_φ are the components of electric field in the spherical coordinate system,

$$\begin{aligned} S_1(\cos\theta) &= \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [a_n \pi_n(\cos\theta) + b_n \tau_n(\cos\theta)] \\ S_2(\cos\theta) &= \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} [a_n \tau_n(\cos\theta) + b_n \pi_n(\cos\theta)] \\ \pi_n(\cos\theta) &= \frac{P_n^1(\cos\theta)}{\sin\theta} \\ \tau_n(\cos\theta) &= \frac{dP_n^1(\cos\theta)}{d\theta}. \end{aligned} \tag{11}$$