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Light radiation pressure upon an optically orthotropic surface

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ABSTRACT

In this paper, we discuss the problem of determination of light radiation pressure force upon an anisotropic surface. The optical parameters of such a surface are considered to have major and minor axes, so the model is called an orthotropic model. We derive the equations for force components from emission, absorption, and reflection, utilizing a modified Maxwell's specular-diffuse model. The proposed model can be used to model a flat solar sail with wrinkles. By performing Bayesian analysis for example of a wrinkled surface, we show that there are cases in which an orthotropic model of the optical parameters of a surface may be more accurate than an isotropic model.

1. Introduction

The theory of light radiation pressure upon space objects is well-established. For celestial bodies, this pressure creates the Yarkovsky acceleration due to uneven heating of their surface [3,4]. There is also a YarkovskyO'KeefeRadzievskiiPaddack (YORP) effect, in which an asteroid can spin-up from emission pressure because of its irregular shape, [5] up to the disintegration of a body [6,7].

For practical applications, the derivation of light radiation pressure force is necessary for the prediction of the dynamics of GNSS satellites [8–13], for interplanetary stations [14–16] and other spacecraft [17].

For solar sail applications, there are many studies of light radiation pressure, including light pressure generalizations [18,19], and special cases – variable reflectance / transmittance coatings [20], degradation effects [21,22], joint analysis of aerodynamic and radiation forces on spacecraft [23], laser propulsion [24,25], transparent sails [26], etc. There are numerous studies of the astrodynamics of solar sails [27–34] etc.

In the space experiments Nanosail-D2 [35], IKAROS [36,37], and LightSail [38] it was shown that any solar sail membrane has general curvature, both regular and semi-random (smoothness), and also small wrinkles.

The light pressure model on the curved solar sail was generalized by Rios-Reyes and Scheeres [39,40], Rios-Reyes [41], Rios-Reyes and Scheeres [42], Scheeres [43], McMahon and Scheeres [44–46] and extended by [47–49]. This model is called the Generalized Sail Model (GSM).

In this paper, we will consider the optical anisotropy from the geometrical sources of this anisotropy. The main sources of this optical anisotropy are wrinkles on the solar sail membrane [50,51]. One special case of the effects of wrinkles on the solar sail efficiency was studied by Greschik [52].

We will derive the equations for light radiation pressure by utilizing the well-established theory of light-matter interaction as in radiative heat transfer [53], and after this, we will move to the vector representation of force.

We will consider the effects of emission, absorption, and reflection on light pressure because further phenomena such as transmission are supposed to be less influential on solar sails than the main effects [18]. For each effect, we will consider both isotropic and anisotropic cases. For reflection, we will utilize Maxwell's reflection model [53], in which we assume that reflection has two components as a sum of diffuse and specular cases with corresponding specularity coefficient s. We will also consider the back reflection phenomenon for the orthotropic model.

2. Reference frames

Let us consider some surface A in Euclidean space with origin O' and associated Cartesian coordinate system $O'x'_1x'_2x'_3$, Fig. 1. We

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Nomenclature

$O'x'_1x'_2x'_3$	global coordinate frame
$Ox_1x_2x_3$	local coordinate frame
$\hat{\mathbf{e}}'_i,\ i=1,2,3$	unit vectors of global coordinate frame
$\hat{\mathbf{e}}_i,\ i=1,2,3$	unit vectors of local coordinate frame
dA	infinitesimal element of surface A
$\hat{\mathbf{n}}$	normal to dA
$\hat{\mathbf{m}}$	orientation vector for orthotropic model (in plane Ox_1x_2)
$\mathbf{r}$	position of dA in global frame
$\theta,\ \beta$	direction angles in local frame
ϵ'_λ	directional spectral emissivity
ϵ'	directional integral emissivity
ϵ	emissivity (for isotropic case)
B	Lambertian coefficient
T	temperature of dA
$\epsilon_1, \epsilon_2, \theta_m$	parameters of orthotropic model for emission
B_m	modified Lambertian coefficient for orthotropic emission
c	speed of light in vacuum
i'^A_λ	directional spectral intensity of irradiation
i'^A	directional integral intensity of irradiation
q_0	integral intensity of light source
$\hat{\mathbf{s}}$	vector from light source to dA
ρ''_λ	bidirectional spectral reflectivity
ρ''	bidirectional integral reflectivity
I'	hemispherical-directional light intensity
s	specularity coefficient
ρ	reflectivity (for isotropic model)
ρ_1, ρ_2, θ_m	parameters for orthotropic model for reflection
B_ρ	modified Lambertian coefficient for orthotropic reflection
$d\mathbf{F}^{Sr}$	fraction of emission pressure in arbitrary direction $\hat{\mathbf{r}}$
$d\mathbf{F}^{Ar}$	fraction of absorption pressure in arbitrary direction $\hat{\mathbf{r}}$
$d\mathbf{F}^{Rr}$	fraction of reflection pressure in arbitrary direction $\hat{\mathbf{r}}^R$
$d\mathbf{F}^S$	light pressure from emission
$d\mathbf{F}^A$	light pressure from absorption
$d\mathbf{F}^R$	light pressure from reflection
$d\mathbf{F}$	total light radiation pressure upon dA

will call this frame a global frame. Let us introduce $\hat{\mathbf{e}}'_i$ – unit vectors for the global frame, $i = 1,\ 2,\ 3$.

On this surface, it is possible to localize an infinitesimal surface element dA for which we introduce a local Cartesian coordinate system $Ox_1x_2x_3$ with unit vectors $\hat{\mathbf{e}}_i$, Fig. 1. The origin of local frame O is situated in the center of dA, and its normal $\hat{\mathbf{n}}$ is equal to $\hat{\mathbf{e}}_3$. $[T]$ is a transformation matrix from the local frame to the global frame. The orientation of Ox_1 and Ox_2 is arbitrary.

In the following equations, for any vector, e.g. $\hat{\mathbf{r}}$, we will use direction angles $(\beta,\ \theta)$ in the local frame as follows (Fig. 2):

- $\beta \in [0,\ \pi/2]$ – angle between vector $\hat{\mathbf{r}}$ and $+x_3$. We consider that the infinitesimal surface element is laying on the plane Ox_1x_2.
- $\theta \in [0,\ 2\pi]$ – angle between axis Ox_1 and a projection of $\hat{\mathbf{r}}$ on the plane Ox_1x_2, counterclockwise around Ox_3.

Direction angles $(\beta,\ \theta)$ may have additional subscripts or superscripts.

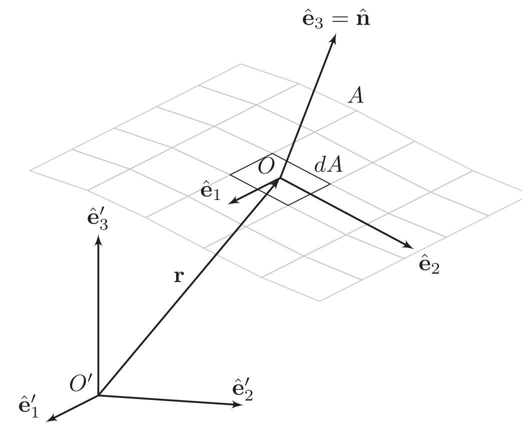

Fig. 1. Definition of coordinate frames.

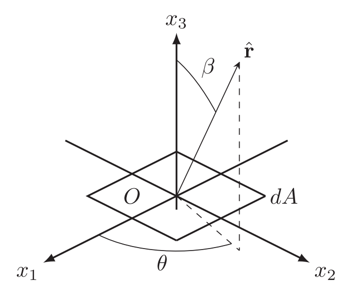

Fig. 2. Definition of angles for arbitrary unit vector $\hat{\mathbf{r}}$.

3. Model

3.1. Thermal emission

Let ϵ'_λ be a directional spectral emissivity, which depends on wavelength, temperature, and shows the difference of emission of dA in direction $(\beta,\ \theta)$ as compared with black body emission in the same direction. One can write the equation of directional integral emissivity [53]:

$$\epsilon'(\beta, \theta, T) = \frac{\pi \int_0^\infty \epsilon'_\lambda i'_{\lambda b} d\lambda}{\sigma T^4},$$

Where σ – Stephan–Boltzmann constant, $i'_{\lambda b}(\lambda, T)$ – spectral intensity of blackbody radiation which is represented by Planck's law:

$$i'_{\lambda b}(\lambda, T) = \frac{2hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda kT}} - 1\right)},$$

Where h – Planck's constant, c – light speed in vacuum, k – Boltzmann constant.

Let us introduce an arbitrary unit vector $\hat{\mathbf{r}}$ in the local frame:

$$\hat{\mathbf{r}}(\beta, \theta) = \sin\beta\cos\theta\hat{\mathbf{e}}_1 + \sin\beta\sin\theta\hat{\mathbf{e}}_2 + \cos\beta\hat{\mathbf{e}}_3,$$

Where $\beta \in [0;\ \pi/2]$ and $\theta \in [0;\ 2\pi]$.

One can write the equation of the fraction of light radiation pressure in direction $\hat{\mathbf{r}}$:

$$d\mathbf{F}^{Sr}(\beta, \theta, T) = -\frac{\epsilon'\sigma T^4}{c}\hat{\mathbf{r}}\cos\beta dA.$$

The superscript S stands for emission ("self"). The relation $\cos\beta dA$ is the area of the infinitesimal element dA under the angle β.

The equation for projection of light pressure force from emission in the direction $\hat{\mathbf{e}}_i$ on the area dA can be written as follows:

$$dF_i^S(T) = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\frac{\pi}{2}} d\mathbf{F}^{Sr} \cdot \hat{\mathbf{e}}_i d\beta d\theta dA. \quad (1)$$