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Iterative discrete ordinates solution of the equation for surface-reflected radiance

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ABSTRACT

This paper presents a new method of numerical solution of the integral equation for the radiance reflected from an anisotropic surface. The equation relates the radiance at the surface level with BRDF and solutions of the standard radiative transfer problems for a slab with no reflection on its surfaces. It is also shown that the kernel of the equation satisfies the condition of the existence of a unique solution and the convergence of the successive approximations to that solution. The developed method features two basic steps: discretization on a 2D quadrature, and solving the resulting system of algebraic equations with successive over-relaxation method based on the Gauss-Seidel iterative process. Presented numerical examples show good coincidence between the surface-reflected radiance obtained with DISORT and the proposed method. Analysis of contributions of the direct and diffuse (but not yet reflected) parts of the downward radiance to the total solution is performed. Together, they represent a very good initial guess for the iterative process. This fact ensures fast convergence. The numerical evidence is given that the fastest convergence occurs with the relaxation parameter of 1 (no relaxation). An integral equation for BRDF is derived as inversion of the original equation. The potential of this new equation for BRDF retrievals is analyzed. The approach is found not viable as the BRDF equation appears to be an ill-posed problem, and it requires knowledge the surface-reflected radiance on the entire domain of both Sun and viewing zenith angles.

1. Introduction

Radiation reflected from the Earth surface presents a valuable source of information about surface properties that can be formalized in the Bi-directional Reflection Distribution Function (BRDF). That information is required to specify a boundary condition for radiative transfer (RT) modeling which is used in aerosol retrievals, cloud retrievals, atmospheric modeling and other applications. Ground based measurements of reflected radiance draw increasing attention as a source of information about anisotropy of surface reflection [1–4], along with development of measurement techniques [5]. Atmospheric correction has to be done to derive BRDF from surface radiance, so retrieval methods were also developed [6,7].

The retrieval methods are based on a comparison of the measured and computed reflected radiance at the ground level. If yet another evaluation of the radiance is needed, then a full radiative transfer problem has to be solved anew for the new guess of BRDF. Decoupling of the atmospheric radiative transfer and anisotropic surface reflectance [8,9] allows one to avoid multiple RT computations if standard problems (no reflection on the boundaries of the atmosphere) are solved. In [8] the solution was found in the form of a series by the number of reflections. In [9] surface-reflected radiance is presented as a solution of an integral equation relating it with BRDF and radiances transmitted through and reflected by the atmosphere. The approaches to solve that equation if standard problems are solved with the discrete ordinates method and spherical harmonics method were also considered in that study.

The first step in both cases is expansion of all functions of the relative azimuth angle into cosine Fourier series and the consequent separation of the problems for the Fourier components. Once they are found, summation of the Fourier series needs to be done. King [10] studied how many terms of the Fourier expansion of the reflection function need to be retained as required in the case of optically thick atmospheres. The observations of that study are: (1) “it is necessary that each term in the Fourier expansion of the phase function satisfy a normalization condition in quadraturized form,” (2) “the reflection function of a semi-infinite atmosphere can be represented by a Fourier series whose upper limit depends strongly on the angles of incidence,” (3) “for aircraft or satellite applications involving scanning radiometers for measuring the reflected intensity field at nadir angles from 0° to 45°, the number of

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terms required in the Fourier expansion of the reflection function for semi-infinite atmospheres will generally not exceed 16," and (4) "Thus in order to maintain a relative accuracy of 0.1% in the reflection function of optically thick atmospheres, more terms may be required in the Fourier series expansion of the reflection function than required for a semi-infinite atmosphere." The last indicates as to the possible increase of the number of terms needed to be retained with the decrease of optical thickness of the atmosphere. In the case of optically thin atmosphere when it makes sense to perform atmospheric correction of the ground measurements, the relative contribution of single scattering prevails over all other orders of scattering. Thus, the number of Fourier components needed is as much as the number of Fourier components of the phase function. Taking into account King's first finding listed above and study [11] stating that "a straightforward numerical evaluation of the Fourier coefficients of sharply peaked phase functions based on the trapezoidal rule has been shown to be more accurate and much more computationally efficient than the use of the Legendre series derived from the addition theorem," the idea of getting rid of Fourier expansions as a first step of the solution of the equation looks promising. This paper proposes an approach to the solution of the integral equation for the surface-reflected radiance based on 2D discretization on a unit hemisphere. A combination of a Gaussian quadrature for integration over cosine of the viewing zenith angle and the regular (equidistant) grid with trapezoidal rule for integration over the relative angle comprises the type of 2D quadrature used in this study.

2. Statement of the problem and notation (equation for the surface reflected radiance)

If a plane parallel scattering medium is illuminated on its top by light coming in direction $\mu_0 = \cos\,\theta_0$, $\phi_0 = 0$, then the diffuse radiance inside the medium and its boundaries is a solution of the radiative transfer equation (RTE):

$$\mu\frac{\partial I}{\partial \tau} + I(\tau,\mu,\phi,\mu_0) = I_0\mu_0 e^{-\tau/\mu_0}\chi(\mu,\mu_0,\phi) + \Lambda\int_0^{2\pi} d\phi' \int_{-1}^{1} d\mu' \chi(\mu,\mu',\phi-\phi') I(\tau,\mu',\phi',\mu_0) \tag{1}$$

where τ is optical depth, Λ – single scattering albedo (SSA), $\mu = \cos\,\theta$, θ, ϕ are the polar and azimuth angles of the direction of propagation of light, $\chi(\mu,\, \mu',\phi-\phi')$ is the scattering phase function normalized with condition:

$$\int_0^{2\pi} d\phi' \int_{-1}^{1} d\mu' \chi(\mu,\mu',\phi-\phi') = 1 \tag{2}$$

RTE (1) has to be supplemented by appropriate boundary conditions (BCs) on the top and bottom boundaries. For further development we need to consider 3 related boundary value problems (BVPs) schematically depicted in Fig. 1(a)–(c). Vertical axis is pointed from top to bottom surface, so that $\mu = \cos\,\theta > 0$ for downward radiation. The first problem is for the atmosphere illuminated from its top and bounded at the bottom by a reflective surface described with BRDF ρ [12]:

$$\begin{aligned} & I(\tau=0,\mu>0,\phi,\mu_0)=0 \\ & I(\tau=\tau_t,\mu<0,\phi,\mu_0)=I_0\mu_0 e^{-\tau_t/\mu_0}\rho(\mu_0,-\mu,\phi) \\ & + \int_0^{2\pi} d\phi' \int_0^1 d\mu'\mu'\rho(\mu',-\mu,\phi-\phi') I(\tau=\tau_t,\mu',\phi',\mu_0) \end{aligned} \tag{3}$$

We will denote this as BVP1. Two other problems are similar: the medium is illuminated from its top and there is no reflection at the bottom surface

$$\begin{aligned} & I(\tau=0,\mu>0,\phi,\mu_0)=0 \\ & I(\tau=\tau_t,\mu<0,\phi,\mu_0)=0 \end{aligned} \tag{4}$$

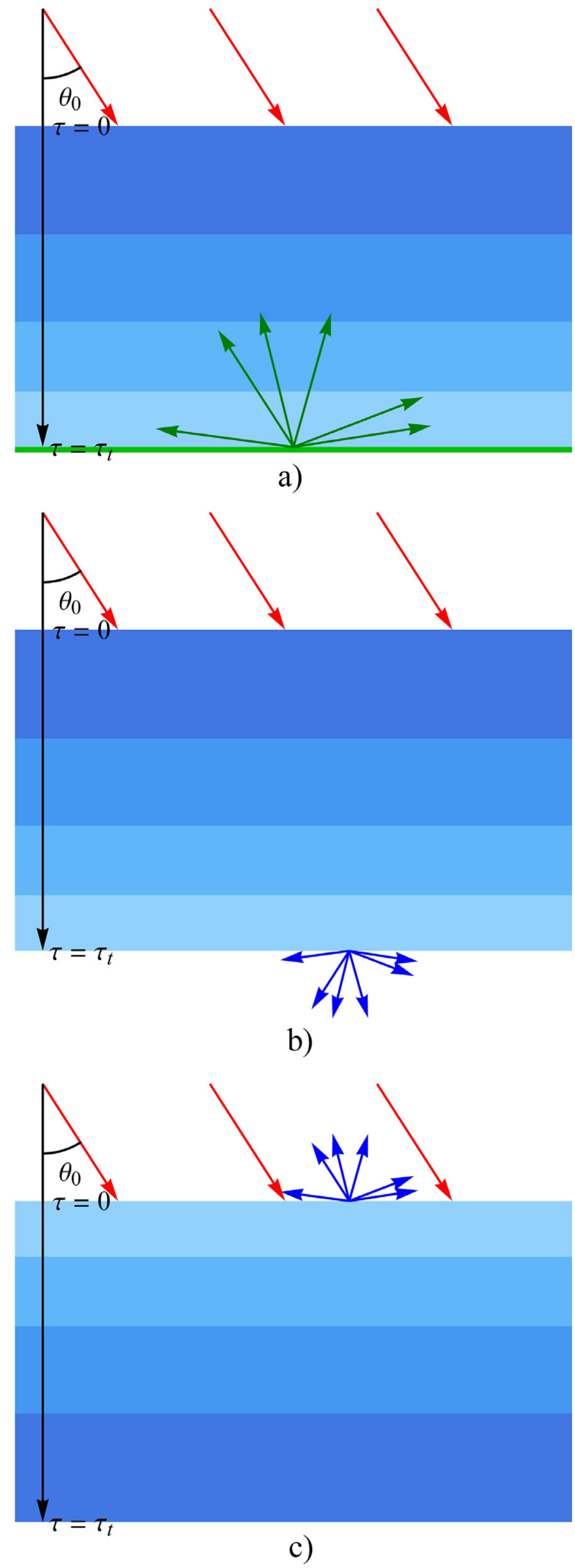

Fig. 1. Schemes of the BVPs: a–1, b–2, c–3. Thick green line in a) depicts reflecting surface. Darker green arrows are for surface-reflected radiance. Blue arrows in b) are for diffuse transmitted radiance. Blue arrows in c) are for radiance reflected by the flipped atmosphere. (For interpretation of the references to color in this figure legends, the reader is referred to the web version of this article.)