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Hyperbolic umbilic caustics from oblate water drops with tilted illumination: Observations

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ABSTRACT

Various groups have reported observations of hyperbolic umbilic diffraction catastrophe patterns in the far-field scattering by oblate acoustically levitated drops with symmetric illumination. In observations of that type the drop's symmetry axis is vertical and the illuminating light beam (typically an expanded laser beam) travels horizontally. In the research summarized here, scattering patterns in the primary rainbow region and drop measurements were recorded with vertically tilted laser beam illumination having a grazing angle as large as 4 degrees. The findings from these observations may be summarized as follows: (a) It remains possible to adjust the drop aspect ratio (diameter/height) $= D/H$ so as to produce a V-shaped hyperbolic umbilic focal section (HUFS) in the far-field scattering. (b) The shift in the required D/H was typically an increase of less than 1% and was quadratic in the tilt. (c) The apex of the V-shaped HUFS was shifted vertically by an amount proportional to the tilt with a coefficient close to unity. The levitated drops had negligible up-down asymmetry. Our method of investigation should be useful for other generalized rainbows with tilted illumination.

1. Introduction

Marston and Trinh discovered that a hyperbolic-umbilic diffraction catastrophe pattern is present in the far field scattering of light from oblate spheroidal water drops viewed in the primary rainbow region [1]. For horizontally illuminated oblate spheroidal water drops, they observed a horizontal V-shaped focal section of a hyperbolic-umbilic caustic when the drop's aspect ratio was 1.305 ± 0.016. The drop was illuminated by a red horizontal laser-beam and the shape of the drop was controlled using acoustic levitation. They reported that for an appropriately located camera, photographs of the drop displayed four-red "patches" associated with scattered rays. Two of the patches were in the plane of the drop's equator while the other two were symmetrically displaced above and below that plane. They described the displaced rays as "skew rays" and noted that the four rays contributed to the catastrophe pattern. Assuming the drop was a perfect spheroid with a relative refractive index of 1.3317, Nye calculated a critical aspect ratio of 1.3114 when the hyperbolic-umbilic focal section appears [2]. Nye's calculation imposed the condition that the skew rays merge with the ordinary Descartes ray in the equatorial plane. There have been several subsequent related investigations of patterns associated with the primary rainbow of oblate drops [3-13]. Observations of the hyperbolic-umbilic focal section have only been reported for horizontally illuminated oblate spheroidal water drops having a vertical symmetry axis. This paper demonstrates the appearance of the pattern for drops with tilted illumination.

This paper investigates the effect of the tilt of the illuminating beam on the critical aspect ratio of the drop at which the hyperbolic-umbilic focal section is observed. It also investigates the effect of the beam's tilt on the vertical and the horizontal location of the V-shaped hyperbolic-umbilic focal section. The results summarized here were previously only available in a dissertation [14]. In all cases investigated here the drops were in the mm size range.

The broader context of the present investigation is that understanding the relationships between waves, wave-fronts, caustics, and scattering spans many fields of investigation [7,15]. Nye's aforementioned approach [2] to analyzing the focal section condition was quickly generalized to predict the location of the cusp diffraction pattern associated with some non-critical aspect ratios of drops [3]. Eventually the V-angle and some related diffraction pattern properties were analyzed [7,10]. The hyperbolic-umbilic diffraction catastrophe focal section requires vanishing outgoing wave-front curvature in the vertical as well as in the horizon-

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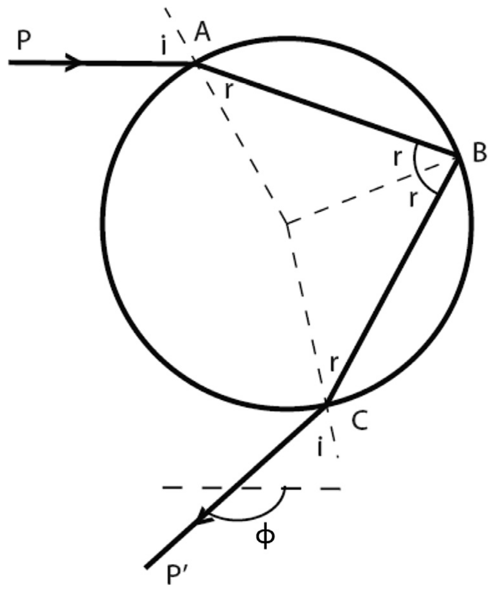

Fig. 1. Top view of the ray diagram of a spheroidal water drop showing the Descartes ray of the primary rainbow. The ray is through the drop in the horizontal equatorial plane. Both the incident ray and the surface normal lie in the horizontal equatorial plane.

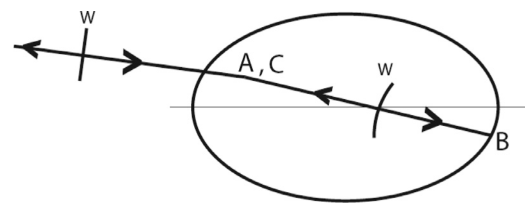

Fig. 2. Qualitative side view illustrating caustic ray path when the hyperbolic-umbilic diffraction catastrophe is observed with tilted illumination. The drop is viewed perpendicular to the horizontal plane. Points A, B, and C, which in Fig. 1 were in the equatorial plane, are now displaced from that plane. Points A and C are on opposite sides of the drop and are on the surface of the drop, displaced from the plane of the elliptical profile.

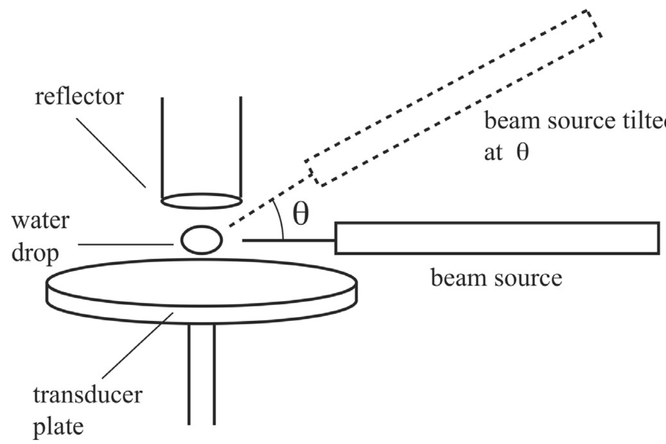

Fig. 3. Experiment arrangement showing the position of the illuminated water drop levitated close to the pressure node nearest the circular transducer plate. The figure is not drawn to scale. The tilt angle was calibrated relative to a horizontal water surface (not shown) using an optical device. The beam source is actually off-set horizontally from the plate as shown in Fig. 4. The plate diameter is 165 mm.

tal plane for a horizontally illuminated drop. The appearance of the caustic is due to the flat wave-front exiting the drop. What were originally referred to as "patches" in images [1] for the rays leaving the drop eventually became known as "glare points" [16] (see Appendix A). The approach of utilizing intrinsically two-dimensional scattering patterns was generalized to characterizing light scattering patterns by non-spherical objects [17,18]. In acoustics it was found that the mathematics of hyperbolic-umbilic caustic wave-fields is useful for predicting observed scattering amplitudes for spheres illuminated by certain caustic wave-fields of sound in water [19].

2. Some considerations from ray theory

Consider first the case of horizontal illumination of an oblate drop. The rays that are necessary for the appearance of the caustic on the primary rainbow region are the once-reflected, twice-refracted, rays shown in Fig. 1. The rays enter through the drop in the horizontal equatorial plane. In Fig. 1, a light ray (P) is incident on the drop. The light enters the drop at A and gets refracted towards the normal of the surface. Upon entering the drop, the ray is deviated by an angle of $i-r$ where i and r are the angle of incidence and the angle refraction respectively. The ray is then reflected inside the drop at the water-air boundary at B, which causes another deviation of 180° – 2r. The ray is then refracted again as it leaves the drop at C with an associated deviation of $i-r$. This gives a total deviation $\phi = 2i - 4r + 180°$ for the ray. The primary rainbow angle is the angle with the least possible deviation for a ray undergoing one internal reflection. Using Snell's law, $\sin i = n_R \sin r$, where n_R is the relative refractive indices of the water drop, the Descartes ray (primary rainbow angle) was found to be near $\phi = 137.69°$ for n_R of 1.33138 and red laser light having a wavelength of 632.8 nm.

Fig. 2 shows the side view of a qualitative ray diagram of an oblate spheroidal water drop with small tilted illumination when the hyperbolic-umbilic diffraction catastrophe is observed. The incident ray enters the water drop, is reflected at B, and then exits the drop by a symmetric path. The hyperbolic-umbilic diffraction catastrophe requires vanishing outgoing wave-front curvature in orthogonal planes. The appearance of the caustic focal section is again expected as the result of a locally flat wave-front exiting the drop. Because of the symmetry required for a flat exiting wave-front, the ray exits the drop with a similar vertical angle it entered the drop with.

For a horizontally illuminated oblate spheroidal drop having a vertical symmetry axis, point B in Fig. 2 lies in the equatorial plane and Nye's analysis is directly applicable. The critical aspect ratio predicted to produce a hyperbolic umbilic focal section in the primary rainbow far-field scattering in that case is [2]:

$$D/H = \left[(3/4)n_R{}^2/\left(n_R{}^2-1\right)\right]^{1/2}. \tag{1}$$

Analytical results for the case of tilted illumination are unavailable. Since the emphasis of the present paper concerns measurements [14], one approach to allowing for tilted illumination based on numerical ray tracing is briefly summarized in Appendix B.

3. Apparatus and method

The experimental method used here is similar to that used in other acoustic-levitation-based studies of rainbow scattering [1,5,9]. A drop of distilled water is acoustically levitated in air forming an oblate spheroidal shape. The drops are typically close to 1.7 mm in diameter. The drop is illuminated with a vertically polarized monochromatic laser beam tilted with respect to the equatorial plane of the drop, and the light scattered by the drop is observed near the primary rainbow angle. The experimental setup for illuminating a levitated water drop is shown in Fig. 3. The aspect ratio of the drop is adjusted by changing the sound amplitude until the hyperbolic-umbilic focal section is observed, at which time