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On the validity of integral localized approximation for on-axis zeroth-order Mathieu beams

A. Chafiq [a], L.A. Ambrosio [b], G. Gouesbet [c], A. Belafhal [a,*]

[a] Laboratory of Nuclear, Atomic and Molecular Physics, Department of Physics, Faculty of Sciences, Chouaïb Doukkali University, P. B 20, El Jadida 24000, Morocco
[b] Department of Electrical and Computer Engineering, São Carlos School of Engineering, University of São Paulo, 400 Trabalhador São-carlense Ave., São Carlos, SP 13566-590, Brazil
[c] UMR 6614/CORIA, CNRS, Université et INSA de Rouen, Site du Madrillet, Avenue de l'Université, BP12 76801 Saint Etienne du Rouvray, France

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ABSTRACT

In a recent paper on radiation pressure forces exerted on a homogenous spherical particle by zeroth-order Mathieu beams (zMBs), the integral localized approximation(ILA) was used to calculate the beam shape coefficients (BSCs) encoding the shape of the beams. Unfortunately, this method is valid only for beams with a propagating factor exp $(\pm ikz)$. In the case of non-diffracting beams the propagation factor is exp $(\pm ik\cos\alpha\, z)$ which involves an extra-cosine term, with α being the axicon angle. Due to this term it has been demonstrated that localized approximations, including ILA, provide a satisfactory description of the intended beam only if the axicon angle is small enough. Zeroth-order Mathieu beams pertain to this type of beams. The present paper is therefore devoted to a comparison between BSCs calculated with an exact procedure and those calculated using ILA, in order to determine a range of validity of the approximate procedure. As a side result, we also establish exact closed-form expressions to the evaluation of BSCs of zMBs.

1. Introduction

The study of interactions between laser light and particles has become a major field of research during the last decades, in particular due to its interest in optical particle characterization, optical trapping and stretching, and morphology-dependent resonances, to name a few [1]. Beside the conventional use of plane waves in the pre-laser ages and of Gaussian beams, many other kinds of beams have been used such as Bessel beams (BBs) [2,3], Laguerre–Gauss beams [4,5], laser sheets [6,7], or parabolic beams [8]. Also, many kinds of particle shapes have been considered by using either Generalized Lorenz-Mie theory (GLMT) [9–11] or the Extended Boundary Condition Method (EBCM) [12]. The description of shaped beams, in these theories, is carried out, implicitly or explicitly, in terms of Vector Spherical Wave Functions (VSWFs) with expansion coefficients expressed in terms of sub-coefficients, usually denoted as $g_{n,TM}^{m}$ and $g_{n,TE}^{m}$, called the TM and TE beam shape coefficients (BSCs), respectively. These coefficients encode the beam shapes.

There exist several methods to evaluate the BSCs, e.g. [13]. When available, the use of an exact analytical formulation with closed-form expressions is to be preferred. The most general technique, actually the original one established in the first steps of development of the GLMT, e.g. [14], is the quadrature technique relying on numerical computations, and available under two different formulations [15], one using a double quadrature over angular coordinates θ and φ (F1-formulation), and the other using a triple quadrature over spherical coordinates r, θ and φ (F2-formulation). Unfortunately, the quadrature methods are time-consuming due to the fact that the kernel to integrate is highly oscillating. A very effective method, however, is the use of a localized approximation which revealed itself to be the most efficient one in the case of Gaussian beams [16,17], recently reviewed in [18,19].

Among the many kinds of beams which have been considered, there has been an increasing interest in non-diffracting beams after the introduction of BBs by Durnin [2], and by Durnin et al. [3]. These beams are non-diffracting (they propagate without any change in their shape) and, furthermore, self-healing. Another class of non-diffracting beams on which we shall focus on this paper is the class of Mathieu beams [20,21,22]. Although BBs, at the present time, are studied more extensively than Mathieu beams, the situation may change in the future because these last beams exhibit an elliptical feature, and a zone of illumination characterized by a central spot and side lobes, which make them appealing for some applications, such as by Kartashov et al. who used Mathieu beams

* Corresponding author.
E-mail address: belafhal.a@ucd.ac.ma (A. Belafhal).

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to produce periodic lattice in order to analyze the properties and stability of two-dimensional optical solitons [23,24]. More specifically, we are concerned with the evaluation of BSCs of linearly x-polarized zMBs.

The problem we now have to address in the present paper is best understood by first considering the case of BBs which share with Mathieu beams the property of having a propagation factor reading as $\exp(\pm ik\cos\alpha\, z)$ in which α is known as the axicon angle (or half-cone angle). This is in contrast with the propagation factor $\exp(\pm ikz)$ used to design localized approximations for "arbitrary shaped beams" in [25]. It has then been demonstrated that the extra $\cos\alpha$ term has a deleterious effect on the quality of localized approximations for BBs (more generally for any kind of beams having a propagation factor containing a $\cos\alpha$-term) [19,26]. The validity of localized approximations in the case of zeroth-order BBs was afterward examined by comparing BSCs obtained by using either an exact closed-form solution [27] or a localized approximation procedure [28]. It was concluded that the localized beam model approximates reasonably well the intended beam only when the axicon angle is typically less than 10° Consequences on the calculation of radiation pressure forces are examined in [29], while discrete superpositions of BBs are considered in [30]. From these complementary works, it is concluded that discrepancies become significant as the axicon angle increases above the paraxial limit, when the BSCs subscript n and superscript m increase or when the incident beam departs from the on-axis case.

The aim of this paper is therefore to provide a similar analysis for Mathieu beams which share, with BBs, the property of having a propagation factor $\exp(\pm ik\cos\alpha\, z)$. BSCs obtained by using closed-form expressions are compared with BSCs obtained by using an integral localized approximation (ILA), in the case of a linearly x-polarized zMB.

The paper is organized as follows. Derivation of analytical expressions of BSCs is presented in Section 2. Also, the convergence of the exact BSC-procedure is verified numerically and compared with the one presented in [31] in the case of zeroth-order BBs. Section 3 provides numerical comparisons between closed-form BSCs and those obtained by ILA, with an emphasis on the influence of some parameters, i.e. axicon angle, ellipticity parameter q, and subscripts m and n as well. Section 4 is a conclusion.

2. Exact calculation of BSCs of *x*-polarized zeroth-order Mathieu beam

A zeroth-order Mathieu beam (zMB) is a solution of the scalar two dimensional Helmholtz equation in elliptic cylindrical coordinates, which can be expressed as a product of radial and angular Mathieu functions [20]. Also, such a beam can be expressed as a sum of BBs of various orders as demonstrated in [21]. It is described in this paper by using a Cartesian coordinate system (O, x, y, z) attached to a point space P, with (ρ_0,φ_0,z_0) being the coordinates of the beam center in this system, and (ρ_g,φ_g,z) the coordinates of P in the beam coordinate system (see Fig. 1). According to Eq. (4) in [21], the scalar zMB can be written as:

$$E = E_0\sum_{j=0}^{\infty}(-1)^j A_{2j}(q)J_{2j}(k_t\rho_g)\cos(2j\phi_g)e^{ik_z z}$$
$$= E_0 e^{ik_z z}\sum_{j=0}^{\infty}\sum_{\nu=\pm 2j}\frac{(-1)^j}{2}A_{2j}(q)J_\nu(k_t\rho_g)e^{i\nu\phi_g}, \quad (1)$$

where $k_t = k\sin\alpha$ and $k_z = k\cos\alpha$ are the transverse and the longitudinal components of the wave vector, respectively, α is the associated axicon angle, q is the ellipticity parameter reading as $q = \frac{h^2k_t^2}{4}$ where h is the interfocal parameter, $J_\nu(.)$ is a Bessel function of first kind and integer order ν and $A_{2j}(q)$ are Mathieu coefficients. We recall also that for $q = 0$, all $A_{2j}(0)$ are zero except for $A_0(0) = 0.7314$. Then, zMBs become zeroth-order BBs.

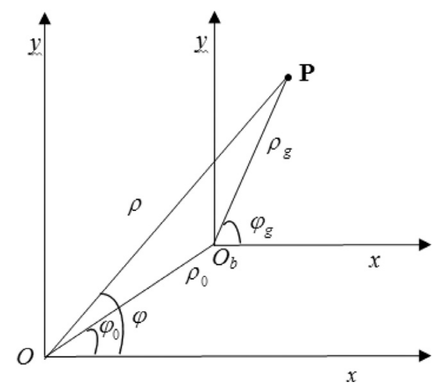

Fig. 1. Cylindrical coordinates of Mathieu beams (of center O_b) and the point P.

Eq. (1) allows us to build scalar or vectorial zMBs from a linear superposition of scalar or vectorial BB solutions of vectorial Helmholtz equation as in [32]. According to Wang et al. [32], the electric and the magnetic fields of Maxwellian x-polarized BBs are given by

$$E_x^b = E_0\cos\alpha J_n(k_t\rho_g)(-i)^n e^{in\phi_g}e^{-ik_z(z-z_0)}, \quad (2.a)$$

$$E_y^b = 0, \quad (2.b)$$

$$E_z^b = -\frac{1}{2}E_0 e^{in\phi_g}e^{-ik_z(z-z_0)}\sin\alpha\Big[(-i)^{n+1}J_{n+1}(k_t\rho_g)e^{i(n+1)\phi_g} + (-i)^{n-1}J_{n-1}(k_t\rho_g)e^{i(n-1)\phi_g}\Big], \quad (2.c)$$

$$B_x^b = B_0\sin^2\alpha\Big[\frac{i\cos^2\phi_g - i\sin^2\phi_g + 2\cos\phi_g\sin\phi_g}{k_t^2\rho_g^2}(n^2-n)J_n(k_t\rho_g) + \frac{in\sin^2\phi_g - in\cos^2\phi_g + 2\cos\phi_g\sin\phi_g}{k_t\rho_g}J_{n+1}(k_t\rho_g) - \cos\phi_g\sin\phi_g J_n(k_t\rho_g)](-i)^n e^{in\phi_g}e^{-ik_z(z-z_0)}, \quad (2.e)$$

$$B_y^b = B_0\Big\{\Big[\sin^2\alpha\frac{\sin^2\phi_g - \cos^2\phi_g + 2i\cos\phi_g\sin\phi_g}{k_t^2\rho_g^2}(n^2-n) + \left(\sin^2\phi_g + \cos^2\alpha\cos^2\phi_g\right)\Big]J_n\left(k_t\rho_g\right) + \sin^2\alpha\frac{\sin^2\phi_g - \cos^2\phi_g - 2in\cos\phi_g\sin\phi_g}{k_t\rho_g}J_{n+1}\left(k_t\rho_g\right)\Big\}\left(-i\right)^n e^{in\phi_g} e^{-ik_z\left(z-z_0\right)}, \quad (2.f)$$

$$B_z^b = B_0\cos\alpha\sin\alpha\left[\frac{\cos\phi_g - i\sin\phi_g}{k_t\rho_g}nJ_n(k_t\rho_g) + i\sin\phi_g J_{n+1}(k_t\rho_g)\right](-i)^n e^{in\phi_g}e^{-ik_z(z-z_0)}, \quad (2.g)$$

in which E_0 and H_0 are the electric and magnetic field strengths, respectively, and where the superscript "b" indicates BBs. Then, the x-polarized zMBs electric field components are

$$\begin{cases} E_x = E_0\cos\alpha\sum_{j=0}^{\infty}\frac{(-1)^j}{2}A_{2j}(q)\sum_{\nu=\pm 2j}J_\nu(k_t\rho_g)(-1)^\nu e^{i\nu\phi_g}e^{-ik_z(z-z_0)} \\ E_y = 0 \\ E_z = -\frac{1}{2}E_0 e^{-ik_z(z-z_0)}\sin\alpha\sum_{j=0}^{\infty}\frac{(-1)^j}{2}A_{2j}(q) \\ \sum_{\nu=\pm 2j}\left[(-i)^{\nu+1}J_{\nu+1}(k_t\rho_g)e^{i(\nu+1)\phi_g} + (-i)^{\nu-1}J_{\nu-1}(k_t\rho_g)e^{i(\nu-1)\phi_g}\right] \end{cases}. \quad (3)$$