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Circularly symmetric frozen waves: Vector approach for light scattering calculations

Leonardo André Ambrosio

Department of Electrical and Computer Engineering, São Carlos School of Engineering, University of São Paulo., 400 Trabalhador são-carlense Ave. 13566-590. Pq. Arnold Schimidt, São Carlos, SP, Brazil

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ABSTRACT

This work introduces particular classes of vector wave fields for light scattering calculations, viz. structured light fields composed of specific superpositions of circularly symmetric Bessel beams of arbitrary order. Also known as *generalized* frozen waves, such beams carry all the non-diffracting properties of their constituents with the additional feature of allowing for an arbitrary design of the longitudinal intensity pattern along the surface of several cylinders of fixed radius, simultaneously. This feature makes the generalized frozen waves especially suitable for optical confinement and manipulation and atom guiding and selection. In the framework of the generalized Lorenz–Mie theory, the beam shape coefficients which describe such beams are evaluated in exact and analytic form, the resulting expressions being then applied in light scattering problems. Particular frozen waves are considered beyond the paraxial approximation, optical forces being calculated for specific dielectric Rayleigh particles.

1. Introduction

Non-diffracting Bessel beams (BBs) are solutions to the vector wave equation that are capable of overcoming the sometimes undesired effects of diffraction over long distances when compared to what is usually called conventional beams [1,2]. Whether generated by finite or annular apertures, axicons, computational holography or so, BBs have proven their value during the last decades in light scattering problems and light-matter interactions, with due attention to optical confinement and manipulation of micro and nano-sized scatterers. In optical tweezers, the multi-ringed structure of BBs can easily provide for simultaneous manipulation of biological particles at multiple planes [3–7].

Even though single arbitrary-order BBs are incapable of providing effective three-dimensional traps, it has recently been suggested that, perhaps and at least in theory, suitably-designed beams constructed from superpositions of BBs could account for that while still (and naturally) carrying all the non-diffracting characteristics of its constituents, viz. resistance to diffraction (and, under certain conditions, to attenuation as well), self-healing and extended focus [8]. The main premise is that an almost arbitrary longitudinal intensity pattern can be modeled by superposing BBs with the same frequency but otherwise distinct longitudinal (or, equivalently, transverse) wave numbers.

Such optical fields - known as *frozen waves* - have been originally conceived as interesting solutions to the scalar wave equation with potential applications in remote sensing, free space communication, optical alignment, optical trapping and atom guiding, to mention a few [9,10]. A finite number of zero-order BBs were initially summed over to create as interesting a longitudinal field pattern as a growing exponential one, even in absorbing media [11,12]. Soon after its conception, natural extensions appeared incorporating not only higher order, but also continuous, generalized, vector and/or finite-energy frozen waves [13–17]. Scalar and vector frozen waves some of which with extremely confined structured fields were theoretically investigated, with the first experimental generations confirming those studies and predictions [18–21].

The first theoretical and numerical considerations of frozen waves for optical trapping and manipulation appeared recently [8,22,23]. However, we observe that the analysis is still quite restricted: scalar frozen waves are taken for granted and are basically transformed into transverse electromagnetic vector beams, thus constraining all subsequent considerations to the paraxial approximation (small half-cone angles). In a sense, that is why the incorporation of frozen waves in the framework of the generalized Lorenz–Mie theories (GLMT) extensions of the Mie theory for arbitrary-shaped beams [24] has been successfully but, at the same time, exclusively carried over by means of the localized approxima-

E-mail address: leo@sc.usp.br

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tion in order to compute the beam shape coefficients (BSCs) which describes the spatial deviations of a particular light field from a plane wave [22,23].

In order to go beyond the paraxial approximation, vector descriptions of frozen waves are due. In the literature, linear, azimuthal, circular, elliptical and radial polarizations have already been considered [16,17], some of which exclusively for discrete frozen waves [16]. However, in view of the multipole expansion for light scattering problems and as interesting as they may be, a more tractable polarization may be considered which allows for analytic descriptions of the corresponding BSCs: the circular symmetric polarization [25,26]. It has been recently shown that exact and analytic expressions of the BSCs for circularly polarized BBs can be found, thus avoiding double and triple integrations (quadrature techniques) [27].

Here, we investigate a promising vector frozen wave for light scattering problems in the context of the GLMT and envisioning its application in the field of optical confinement and manipulation, viz., the circularly symmetric frozen wave. In addition to introducing a full Maxwellian nature to such diffractionless beams, the discrete frozen waves here considered may be constructed from an arbitrary number of discrete frozen waves of different order. They are, therefore, *generalized* frozen waves (GFWs) of multiple order [15]. The advantage of dealing with GFWs should be evident: they automatically provide means to construct independent and simultaneous longitudinal intensity patterns along specific distances [15]. Besides, as noticed elsewhere, GFWs may be seen as hollow optical beams in atom guiding [15].

We divide this paper as follows. Section 2 presents a background on the theoretical aspects of GFWs and their circularly symmetric description in the GLMT in terms of analytic expressions for the BSCs. In Section 3, two examples of circularly symmetric GFWs are considered with longitudinal intensity patterns which may be of interest, e.g. in optical trapping and manipulation of micro-particles. Radiation pressure forces are calculated for specific Rayleigh scatterers in order to infer the capabilities of GFWs in providing effective three-dimensional traps. Finally, our conclusions are presented.

2. Theoretical background

In order to give a brief account on scalar GFWs, let us first consider the original frozen wave constructed from $2N+1$ scalar BBs of arbitrary order ν. Using cylindrical coordinates (ρ, ϕ, z) and with the time-harmonic factor $exp(+i\omega t)$ implicitly assumed, one has the discrete scalar frozen wave [12]:

$$\psi_{FW}(\rho,\phi,z)=\sum_{q=-N}^{N}A_q J_\nu\left(k_{\rho q}\rho\right)e^{i\nu\phi}e^{-ik_{zq}z}. \tag{1}$$

In (1), A_q is the complex coefficient and $k_{\rho q}$ (k_{zq}) is the transverse (longitudinal) wave number of the q-th BB. $J_\nu(.)$ is a Bessel function of first kind and order ν (ν integer). Any longitudinal intensity pattern (LIP) of interest, $|\psi(\rho=0,z)|^2=|F(z)|^2$, may be specified by a judicious choice of the coefficients A_q for $\nu=0$, as thoroughly exposed in the literature (notice that, for each choice, the diffraction limit must be respected). The resulting LIP may be shifted from the optical axis by increasing the order ν [10,12]. Only forward propagating BBs are to be included in (1).

A scalar (and discrete) GFW may be constructed from (1) by superposing frozen waves of different orders. One then writes, with a slight change in notation with respect to previous works [15],

$$\psi_{GFW}(\rho,\phi,z)=\sum_{\nu=-\infty}^{\infty}\sum_{q=-N}^{N}B_\nu A_{q\nu}J_\nu\left(k^\nu_{\rho q}\rho\right)e^{i\nu\phi}e^{-ik^\nu_{zq}z}, \tag{2}$$

where $B_\nu=1/[J_\nu(.)]_{\max}$ are weighting coefficients and

$$\begin{aligned} k^\nu_{zq}&=Q_\nu+\frac{2\pi}{L}q,\\ k^\nu_{\rho q}&=\sqrt{k^2-\left(k^\nu_{zq}\right)^2},\\ A_{q\nu}&=\frac{1}{L}\int_0^L F_\nu(z)e^{i\frac{2\pi}{L}qz}dz. \end{aligned} \tag{3}$$

In (2) and (3), the coefficients $A_{q\nu}$ for a specific value of ν are calculated from the intended LIP $|F_\nu(z)|^2$ which is to be formed along the range $0\le z\le L$ (or, alternatively, $-L/2\le z\le L/2$) at a radial distance given approximately by the first non-null root of $\left|J'_\nu\left(\rho\sqrt{k^2-Q_\nu^2}\right)\right|$, the prime indicating derivative with respect to the argument. For $\nu=0$, the spot radius r_0 may be calculated from the relation $r_o^2=\left(k^2-2.4^2\right)/Q_0^2$ [12]. Examples of scalar GFWs can be found, e.g. in [15]. For our purposes and considering that all normalization issues may be transferred to $F_\nu(z)$, we henceforth take $B_\nu=1$.

2.1. Circularly symmetric vector GFWs

The scalar GFW as described in (2) is of practical interest in light scattering experiments only when the paraxial approximation conditions are satisfied. This is because practical BBs with high half-cone angle are of an intrinsic vector nature. Therefore, solutions to the scalar wave equation must be reinterpreted as components or specific terms of such components of a vector field or potential. Non-paraxial beams must be expressed in vector form, and scalar fields like (2) must be incorporated into the GLMT with the due care.

During the last years, scalar frozen waves have been introduced into the GLMT by taking the paraxial approximation for granted [8,22]. The scalar field, (1), was taken to be a particular transverse electric field component, the vector description considering solely a corresponding transverse magnetic field. At least for single BBs, it has been shown that localized approximations are well suited for calculating the BSCs [28–33], and one can expect this to hold also true for scalar frozen waves, thus providing analytic expressions for them while still preserving good numerical accuracy. However, once the BSCs are calculated, the GLMT automatically remodels the original light field so as to turn it into a true Maxwellian field. The final vector field, on the other hand, may not correctly correspond to the vector field expected when starting with vector BBs from the outset. In fact, discrepancies will be more pronounced for highly non-paraxial beams [32,33].

From the above considerations, it is of theoretical, numerical and practical interest to construct vector GFWs for light scattering calculations. This demands vector BBs as their constituents. In view of that, we ask ourselves how could we construct a vector GFW which, at the same time: (i) may be composed of non-paraxial vector BBs; (ii) is well suited for theoretical, numerical and practical studies in the field of optical confinement and manipulation; (iii) can have its BSCs put into analytical and (iv) exact form. Davis and aplanatic vector GFWs appears as the most promising candidates.

A general description of circularly symmetric Bessel beams is found in [26]. Four polarizations are considered, but here we shall focus our attention on two of them, i.e., on linear polarizations (1,0) and (0,1). The other two are linear combinations of those here considered, being reminiscent of circular polarization. The field components of a circularly symmetric vector GFW with polarization (1,0) may be found by performing (i) a discrete superposition of vector BBs with the same order and polarization, thus generating vector frozen waves of arbitrary but single order, and (ii) a sum of the previous frozen waves, each of which with a specific (and possibly different) order. One then finds, after some straight-