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Optical depth in particle-laden turbulent flows

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ABSTRACT

Turbulent clustering of particles causes an increase in the radiation transmission through gas-particle mixtures. Attempts to capture the ensemble-averaged transmission lead to a closure problem called the turbulence-radiation interaction. A simple closure model based on the particle radial distribution function is proposed to capture the effect of turbulent fluctuations in the concentration on radiation intensity. The model is validated against a set of particle-resolved ray tracing experiments through particle fields from direct numerical simulations of particle-laden turbulence. The form of the closure model is generalizable to arbitrary stochastic media with known two-point correlation functions.

1. Introduction

Radiation transport through particulate media is a common natural and industrial phenomenon, with examples appearing in atmospheric physics [1], sooting flames [2], fire protection systems [3], and solar energy systems [4,5]. In these examples radiation can be the dominant mode of heat transfer, and modeling the heat transfer correctly becomes crucial to understanding the dynamics of these systems. If the particles are uniformly random in space, then the particle distribution may be approximated as Poissonian and models based on the radiative transfer equation become available to compute the heat transfer. However, in particle-laden flows, the distribution is rarely Poissonian, and the resulting radiation transport will differ from classic transport theory. This phenomenon is referred to as the turbulence-radiation interaction (TRI), and it is crucial to predicting mean heat transfer and temperature fields in turbulent flows with strongly coupled radiation [2,6,7]. At high temperatures, the blackbody emission coupled with fluctuations in flow velocity and density produces the emission-TRI, which modulates the emission losses in a radiating flow. At lower temperatures, fluctuations and correlations in a medium density produce the absorption-TRI, which affects the mean opacity and radiation absorption in a flow. In this work we are concerned with the physics and modeling of the absorption-TRI.

In turbulent flows, the particle positions may become positively correlated. In these cases, the rate of radiation attenuation decreases, effectively increasing the optical depth in the particle cloud [8–10] or leading to sub-exponential attenuation. In flows of settling or precipitating particles, the particle positions become negatively correlated, and radiation undergoes super-exponential attenuation [11]. The question of modeling transport physics in correlated random media is a challenge, and a number of different strategies have been proposed to approximate the ensemble-averaged radiation attenuation rate law.

The most general approach would be to derive a set of governing equations to model the behavior of correlated media directly. [12] derived a generalized Boltzmann transport equation to allow for correlation in scattering events. The resulting equation adds a time-like variable to track the distance between collisions which must be integrated in an iterative approach, and is thus more computationally intensive. An alternative method was developed by [13] to handle arbitrary power-law decay rates via a Markov Chain approach to solve for average radiation attenuation. A well-known method to handle binary stochastic media was given by [14], who derived a system of coupled transport equations using renewal theory that are exact in the limit of purely absorbing media. The most general and rigorous method to solve for radiation transport is to directly resolve the electromagnetic fields in the correlated particle fields [15,16] using integral equation methods. While these approaches are promising, they also add a great deal of complexity to the solution of the transport equation.

An alternative approach is to seek approximate radiative properties of the correlated medium that yield the same ensemble-averaged intensity as the random medium. [17] developed a rescaled transport equation to match an asymptotic analysis of a non-local transport equation. [18,19] relate the rate of radiation attenuation to the statistical properties of the correlated medium, using moment expansions and spectral analysis of the absorption

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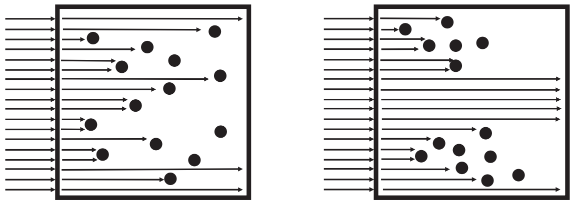

Fig. 1. Poisson-distributed particles (left) are more effective than turbulent-clustered particles (right) at blocking radiation from transmitting through space.

coefficient structure function. [9] demonstrated the general form of a radiation attenuation rate law in arbitrary polynomial expansions, and [11] demonstrated an explicit connection between the two-point correlation function and the effective opacity of a particulate medium. While these approaches may be inherently less accurate than solving the more general governing equations for stochastic media, their relative simplicity makes them an attractive choice for inclusion in existing transport codes and coupled flow-radiation solvers.

When it comes to predicting radiation transport in particle-laden turbulent media, a complicating factor is the coupling between the particle field and a turbulent flow. It is well known that particles form clusters due to interactions with turbulence [20,21], creating highly non-uniform particle fields. The controlling parameter for this clustering is the Stokes number $\mathrm{St} = \tau_p/\tau_\eta$, measuring the ratio of the particle aerodynamic relaxation time τ_p and the Kolmogorov time scale τ_η of the turbulence. When $\mathrm{St} \sim O(1)$, the particle clustering is maximized [20]. As depicted in Fig. 1, these non-uniform particle fields may be less effective at absorbing radiation globally. Frankel et al. [22] showed that the resulting particle fields can cause up to 15% reduction in the effective optical depth of the system. While it is possible to model these fields as inhomogeneous Poisson processes with a localized particle concentration, the definition of concentration is somewhat ambiguous and can lead to numerical difficulties [22]. Instead, one might consider a direct model for radiation transport through turbulent particle-clusters to avoid requiring knowledge of the instantaneous particle fields, similar to the development of Reynolds-averaged models for turbulent flows [23].

In this work we will take such an approach to develop a simple closure for the particle-turbulence-radiation interaction problem. Our interest is in predicting the effect of particle clustering on the opacity of a medium. To do so, we will first develop a general model for the decrease in opacity based on particle spatial distributions, and then apply it to the specific case of particle-laden turbulence, where the particles are smaller than the Kolmogorov length scale of the turbulence. The model is then validated against particle-resolved ray tracing calculations for realizations of particle clusters computed from direct numerical simulations of particle-laden turbulence. To aid in the model development, we restrict our attention to absorbing spheres in the geometric optics limit at dilute volume fractions.

2. Closure model

2.1. Two-point correlations

The development of our closure model relies on the two-point correlation function for the particle number density. In order to derive this quantity, we make use of the particle radial distribution function (RDF). The RDF $g(r)$ is related to the probability of finding a particle some distance r away from a central particle. The differential RDF $h(r) = g(r) - 1$ is also sometimes used for numerical convenience. Previous studies have considered semi-empirical models of the RDF as a function of the particle Stokes number and Kolmogorov length scale. In this study, we make use of the form provided in [24], which gives the RDF approximately as

$$h(r) = c_0\left(\frac{r}{\eta}\right)^{-c_1}\exp\left(-\frac{c_2 r}{\eta}\right) \tag{1}$$

where η is the Kolmogorov length, and c_0, c_1, and c_2 are model coefficients given by

$$c_0 = \frac{\mathrm{Re}_\lambda}{\mathrm{Re}_{\lambda o}}\frac{7.92\mathrm{St}^{1.80}}{0.58+\mathrm{St}^{3.29}} \qquad c_1 = \frac{0.61\mathrm{St}^{0.88}}{0.33+\mathrm{St}^{2.38}}$$
$$c_2 = 0.25 \tag{2}$$

where $\mathrm{St} = \tau_p/\tau_\eta$ is the Stokes number, measuring the ratio of the particle aerodynamic relaxation time τ_p and the Kolmogorov time scale τ_η. $\mathrm{Re}_\lambda = u_{rms}\lambda/\nu$ is the Reynolds number based on the Taylor microscale λ with root-mean-square velocity u_{rms} and fluid kinematic viscosity ν. Following [24] we pick $\mathrm{Re}_{\lambda o} = 54.5$. Eq. (1) was fit against a DNS database for particles without momentum or energy coupling to the flow, and thus we expect this formula to perform well in dilute particle mixtures where effects of particle two-way momentum and thermal coupling are relatively small [24–26]. It is worth noting that this Reynolds number dependence saturates at values higher than $\mathrm{Re}_\lambda \sim 100$ [27], but in the regime of interest this linearity is accurate.

It can be shown that in homogeneous and isotropic systems, the RDF may also be expressed in terms of particle number density as [28,29]

$$g(r) = \frac{\langle n(\vec{r}_o)n(\vec{r}_o + r\hat{s})\rangle}{\langle n\rangle^2} \tag{3}$$

where n is the particle number density, $\vec{r}_o$ is any location in the domain, and $\hat{s}$ is a unit direction vector. The brackets denote the ensemble average operator. The two-point correlation function for number density may then be computed as

$$\Phi_{nn}(r) = \langle n'(\vec{r}_o)n'(\vec{r}_o + r\hat{s})\rangle = \langle n\rangle^2 h(r) \tag{4}$$

where $n' = n - \langle n\rangle$ is the fluctuating component of the number density.

For the radiation model development, it is straightforward to express the two-point correlation for the spatially varying extinction coefficient field in terms of correlations in local number density given their proportional dependence as

$$\sigma = \frac{\pi d^2}{4}n. \tag{5}$$

For a particle-gas mixture with monodisperse particle diameter d one can write the two-point correlation function of the extinction coefficient as

$$\Phi_{\sigma\sigma}(r) = \left(\frac{\pi d^2}{4}\right)^2\Phi_{nn}(r). \tag{6}$$

2.2. Turbulent extinction coefficient

We now turn to the closure problem itself. For cold and absorbing particles, the equation of radiative transfer reduces to the Bouguer-Beer (BB) law:

$$\hat{s}\cdot\nabla I = -\sigma(\vec{r})I \tag{7}$$

where $\hat{s}$ is the direction vector and $I = I(\vec{r}, \hat{s})$ is the radiation intensity. Without loss of generalization, we restrict our attention to radiation transmission in the $+x$ direction, and assume an initial intensity $I(x=0) = I_o$:

$$\frac{dI}{dx} = -\sigma(x)I \qquad I(x=0) = I_o. \tag{8}$$

In homogeneous and isotropic turbulence, the mean value of σ is constant, but there are spatial fluctuations at any given instant.