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Transmittance of semitransparent windows with absorbing cap-shaped droplets condensed on their backside

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ABSTRACT

This study aims to investigate systematically light transfer through semitransparent windows with absorbing cap-shaped droplets condensed on their backside as encountered in greenhouses, solar desalination plants, photobioreactors and covered raceway ponds. The Monte Carlo ray-tracing method was used to predict the normal-hemispherical transmittance, reflectance, and normal absorptance accounting for reflection and refraction at the air/droplet, droplet/window, and window/air interfaces and absorption in both the droplets and the window. The droplets were monodisperse or polydisperse and arranged either in an ordered hexagonal pattern or randomly distributed on the backside with droplet contact angle θ_c ranging between 0 and 180° The normal-hemispherical transmittance was found to be independent of the spatial distribution of droplets. However, it decreased with increasing droplet diameter and polydispersity. The normal-hemispherical transmittance featured four distinct optical regimes for semitransparent window supporting nonabsorbing droplets. These optical regimes were defined based on contact angle and critical angle for internal reflection at the droplet/air interface. However, for strongly absorbing droplets, the normal-hemispherical transmittance (i) decreased monotonously with increasing contact angle for $\theta_c < 90°$ and (ii) remained constant and independent of droplet absorption index k_d, droplet mean diameter d_m, and contact angle θ_c for $\theta_c \geq 90°$ Analytical expressions for the normal-hemispherical transmittance were provided in the asymptotic cases when (1) the window was absorbing but the droplets were nonabsorbing with any contact angles θ_c, and (2) the droplets were strongly absorbing with contact angle $\theta_c > 90°$ Finally, the spectral normal-hemispherical transmittance of a 3 mm-thick glass window supporting condensed water droplets for wavelength between 0.4 and 5 μm was predicted and discussed in light of the earlier parametric study and asymptotic behavior.

1. Introduction

Several experimental measurements and numerical simulations have investigated the effects of water droplets condensed on the backside of glass or plastic windows on light transfer [1–8]. In general, dropwise condensation was found to decrease the transmittance of wet window for both visible and infrared radiation except for anti-drop (hydrophilic) surfaces [2–4]. In the photosynthetically active radiation (PAR) region (400–700 nm) - relevant to greenhouses [7] and microalgal culture systems [9] - reduction in transmittance is mainly due to the total internal reflection at the water droplet/air interfaces [1,10]. In the infrared region - relevant to solar desalination systems [11] - absorption by glass or plastic windows and water droplets plays a significant role in the transmittance reduction [6].

Recently, we investigated systematically the effects of nonabsorbing cap-shaped droplets condensed on the backside of transparent windows on their directional-hemispherical transmittance [10]. The latter was found to be independent of the size and spatial distributions of the droplets. Four optical regimes were identified in the normal-hemispherical transmittance as a function of contact angle θ_c and defined with respect to the critical angle θ_{cr} for total internal reflection at the droplet/air interface. The directional-hemispherical transmittance was nearly constant for droplet contact angle θ_c either smaller than the critical angle θ_{cr} (Regime I) or larger than 180°-θ_{cr} (Regime IV). However, for θ_c between θ_{cr} and 180°-θ_{cr}, the normal-hemispherical transmittance decreased rapidly with increasing θ_c to reach a minimum at 90° (Regime II) and increased rapidly with increasing θ_c up to 180°-θ_{cr} (Regime III). In addition, the normal-hemispherical transmittance decreased monotonously with increasing droplet projected surface area coverage for contact angle larger than θ_{cr}. However, for $\theta_c < \theta_{cr}$ (Regime I), the normal-hemispherical transmittance increased

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slightly with increasing droplet projected surface area coverage. Finally, experimental measurements and numerical simulations of light transfer in transparent or semitransparent windows supporting nonabsorbing or absorbing droplets were reviewed in details in our recent study [10] and need not be repeated.

The present paper extends our previous study [10] to situations when the condensed cap-shaped droplets and/or the window are absorbing. Here also, the effects of (i) droplets' spatial arrangement and (ii) size distribution, (iii) incident angle, (iv) complex index of refraction of the window and (v) the droplets, and (vi) contact angle were systematically investigated. The results could be used for the design, material selection, and thermal management of greenhouses, solar desalination, and photobioreactor systems.

Nomenclature

A	absorptance
d	droplet diameter, μm
d_m	mean diameter of droplets, μm
d_p	projected diameter of droplets, μm
f_A	droplet projected surface area coverage
f_N	frequency distribution of photons having passed through N droplets before being transmitted
H	thickness of the window, mm
j	imaginary unit
k	absorption index
l	path length of photon bundles, mm
l'	probable path length of photon bundles, mm
l_c	capillary length, μm
L	length of the window, mm
m	complex index of refraction
M	number of photon bundles
n	refractive index
$\mathbf{n}$	unit normal vector
N	effective refractive index
$\mathbf{p}$	position vector of photon bundles
r	reflection coefficient
R	reflectance
R_l	random number
$\mathbf{s}$	unit vector of photon bundle directions
W	width of the window, mm
Greek symbols	
θ_c	contact angle, °
θ_{cr}	critical angle, °
φ	azimuthal angle, °
λ	wavelength, μm
ρ	interface reflectance
κ	absorption coefficient, μm^{-1}
$\tilde{\kappa}$	effective absorption coefficient, μm^{-1}
τ	transmissivity
σ	standard deviation of droplet diameter, μm
Subscripts	
i	incident
r	reflected
t	transmitted
a	air
w	window
d	droplet
f	film
dh	directional-hemispherical
nh	normal-hemispherical
λ	spectral
$\perp$, $\parallel$	perpendicular and parallel polarized radiation

2. Analysis

2.1. Problem statement

Figs. 1(a) and 1(b) show the top and side views of the simulated polydisperse droplets randomly distributed on the back window with length L, width W, and thickness H. The window was exposed to collimated monochromatic radiation of wavelength λ incident on its front face at a polar angle θ_i. Photons were reflected, transmitted, or absorbed by the semitransparent window with refractive and absorption indices denoted by n_w and k_w, respectively. Part of the incident radiation was transmitted through the back face of the window into the droplets of refractive and absorption indices denoted by n_d and k_d, respectively. Then, radiation could be (i) internally reflected within the droplet, (ii) transmitted through the droplet/air interface or the droplet/window interface, or (iii) absorbed by the droplets. In the present study, the dimensions of the window for simulating randomly distributed monodisperse or polydisperse droplets were $L = W = 5$ mm, and $H = 3$ mm. The refractive and absorption indices of the surrounding air were taken as $n_a = 1.0$ and $k_a = 0$.

2.2. Assumptions

To make the problem mathematically tractable, several assumptions were made in our simulations:

1. All interfaces were optically smooth so that all reflections were specular and the generalized Snell's law and Fresnel's equations were valid.
2. The droplets were cap-shaped with a constant curvature and diameter d much smaller than the capillary length l_c, i.e., $d \ll l_c$ [12]. For water droplets in air, l_c equals 2.7 mm [10]. Here, the droplets diameter d was smaller than 270 µm to satisfy $d \ll l_c$ [10].
3. The dimensions of the window and droplets were much larger than the radiation wavelength so geometric optics prevailed and interferences and other wave effects could be ignored.
4. The boundary conditions on the sides of the window were periodic.

The generalized Snell's law for reflection and refraction at the interface between two adjacent absorbing media, referred to by subscript "i" (incident side of the interface) and "t" (transmitted side) with respective complex index of refraction $m_i = n_i - \mathrm{j}k_i$ and $m_t = n_t - \mathrm{j}k_t$ where $\mathrm{j}^2 = -1$, can be written as [13–15]

$$N_i \sin\theta_i = N_i \sin\theta_r = N_t \sin\theta_t. \tag{1}$$

Here, θ_i, θ_r, and θ_t are respectively the angles of the incident, reflected, and transmitted radiations defined according to

$$\mathbf{s}_r = \mathbf{s}_i - 2\cos\theta_i \mathbf{n}, \tag{2}$$

$$\mathbf{s}_t = \mathbf{s}_i N_i / N_t + (\cos\theta_t - \cos\theta_i N_i / N_t)\mathbf{n}. \tag{3}$$

The unit vectors $\mathbf{s}_i$, $\mathbf{s}_r$, and $\mathbf{s}_t$ represent the incident, reflected, and transmitted directions while $\mathbf{n}$ is the unit normal vector to the interface pointing towards the transmitted medium (see Fig. S1 in Supplementary Material). The effective refractive indices N_i and N_t of the media on the incident and transmitted sides of the interface are respectively defined as [13–15]

$$N_i^2 = \frac{1}{2}\left[\sqrt{\left(n_i^2 - k_i^2\right)^2 + \left(2n_i k_i / \cos\theta_i\right)^2} + \left(n_i^2 - k_i^2\right)\right], \tag{4}$$