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Analysis of forward scattering of an acoustical zeroth-order Bessel beam from rigid complicated (aspherical) structures

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ABSTRACT

The forward scattering from rigid spheroids and endcapped cylinders with finite length (even with a large aspect ratio) immersed in a non-viscous fluid under the illumination of an idealized zeroth-order acoustical Bessel beam (ABB) with arbitrary angles of incidence is calculated and analyzed in the implementation of the *T*-matrix method (TTM). Based on the present method, the incident coefficients of expansion for the incident ABB are derived and simplifying methods are proposed for the numerical accuracy and computational efficiency according to the geometrical symmetries. A home-made MATLAB software package is constructed accordingly, and then verified and validated for the ABB scattering from rigid aspherical obstacles. Several numerical examples are computed for the forward scattering from both rigid spheroids and finite cylinder, with particular emphasis on the aspect ratios, the half-cone angles of ABBs, the incident angles and the dimensionless frequencies. The rectangular patterns of target strength in the (β, θ_s) domain (where β is the half-cone angle of the ABB and θ_s is the scattered polar angle) and local/total forward scattering versus dimensionless frequency are exhibited, which could provide new insights into the physical mechanisms of Bessel beam scattering by rigid spheroids and finite cylinders. The ray diagrams in geometrical models for the scattering in the forward half-space and the optical cross-section theorem help to interpret the scattering mechanisms of ABBs. This research work may provide an alternative for the partial wave series solution under certain circumstances interacting with ABBs for complicated obstacles and benefit some related works in optics and electromagnetics.

1. Introduction

During the past decades, study on acoustic scattering has attained a great development and attracted increasing attention from investigators all over the world. As reported in the literature, there is a wide range of applications for acoustic scattering in practical and potential engineering fields, such as non-destructive testing technology, ultrasonic medical diagnostic imaging technology, sound navigation and ranging (SONAR), sound detection and ranging (SODAR), acoustic Doppler current profiler, acoustic tweezers/torques/levitation devices, and so forth. Specifically, the monostatic and bistatic sonars are designed and manufactured based in part on acoustic scattering theories. The main difference between the monostatic and bistatic sonar devices is the spatial location distribution of acoustic source and receiver, namely, the acoustic source and receiver of the bistatic sonar are spatially separated while those of the monostatic are not. Therefore, it is obvious to note that the monostatic sonar benefits a lot from research work on backscattering, in contrast for the bistatic sonar, one of its special case (i.e. forward scattering) is widely used in engineering practices since it can provide a large acoustic gain in the forward direction around the target. As claimed in Refs. [1–3], scattering strength in the forward direction is generally stronger than that in the backward scattering. Furthermore, the forward scattering of target is less sensitive to the incidence angle of sound than backscattering which could be applied to calculate scattering from moving targets stably and accurately. Based on the above statements, it is therefore significant and necessary to make further investigations on the forward scattering of a variety of aspects, which may in turn improve the forward scattering theories and benefit several potential applications and engineering practices.

However, most of the published work on the forward scattering in the literature concentrated on the ordinary plane wave incidence case [1–8]. When it comes to *acoustical Bessel beams* (ABBs), the relevant research on forward scattering is rarely found and the published works in numerical computation were mainly focused on spherical shapes [9,10]. As extensively studied both in optical

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[11–15] and acoustical fields [16–22], Bessel beams are demonstrated to be one kind of undistorted waves, having shown its own advantages over the plane waves and Gaussian beams for its novel characteristics including the non-diffraction property [23–26] and self-construction ability [27–29]. In general, ABBs are roughly classified into two kinds: the zeroth-order and high orders Bessel beams, with their amplitude profiles proportional to the zeroth-order Bessel function J_0 and higher order Bessel function J_m (m is an integer with $m \geq 1$), respectively. Due to the properties of Bessel functions, it is thus reasonable to find that the zeroth-order ABB has a maximum in amplitude at its center of cross-section profile with concentric rings of decreasing radial amplitude and axisymmetric azimuthal phase, while the higher orders ABBs possess an axial null in amplitude and has an azimuthal phase gradient [17]. It may be helpful to the readers that the form function of the scattering of higher orders ABBs could be deduced as a special case of those of the nondiffracting Lommel beams [30]. Besides, it should be noted that the fractional Bessel beams are not proper solutions of the scalar Helmholtz equation because of the phase discontinuity when the azimuthal angle is ϕ=0 [31]. Intuitively, the acoustic radiation induced by progressive plane wave scattering tends to push the scatterers away from sources in the propagation direction of the beam, which is here termed as positive radiation force (relative to negative radiation force). Due to the aforementioned characteristics, however, this is not always the case for ABBs. Because of the peculiar features different from those of ordinary plane waves, negative acoustic radiation force [16,32] and even acoustic radiation torque [19,33] could be realized in the context of a single ABB under certain circumstances, which may provide impetus to design acoustic tweezers and torque devices in area of acoustic manipulation and control (pulling, pushing, rotating or levitating particles).

In the recent past decade, the problems of ABBs have been studied by means of analytical, numerical and experimental approaches. However, prior studies using analytical (partial wave series solution) [9,16] and experimental methods [21,22] apparently had their emphasis on the spherical obstacles. For the partial wave series method (PWSM), several principles and novel phenomena of backscattering and bistatic scattering for spherical shapes have been reported [9,16–19]. From the computational standpoint, however, it seems that the PWSM in spherical coordinate frame may fail for the large-aspect-ratio aspherical shapes, such as spheroid and finite cylinder with endcaps. This is due to the fact that the spherical harmonics are employed as the basis functions of expansion and the system of linear equations may become unsolvable because of an ill-condition during matrix inversion procedures [34,35]. When it comes to experimental investigations, the set-ups for producing Bessel beams and receiving scattering signals are usually very expensive and moreover, the generating Bessel beams are not idealized because of the finite power of the sources. In addition, the experiment setup still suffers some technique problems, overheating of transducers for instance [20–22]. Based on the above descriptions, it is therefore reasonable and necessary to introduce the T-matrix method (TMM) into the computational area for ABBs since the TMM (a semi-analytical and semi-numerical approach) is a versatile tool to deal with scattering in acoustic [36–42], electromagnetic [43–45] and elastic [46,47] fields. In addition, the TMM has been extended to calculate the acoustic scattering under the illumination of Bessel beams interacting with elastic spherical shapes [48] and spheroids [49].

Nevertheless, to the best of the authors' knowledge, systematic research on the forward scattering of ABBs is somewhat limited, especially for aspherical targets. As is known to all, the versatile TMM was proved to show good convergence and high precision especially for axisymmetric shapes, including spherical and aspherical, rigid and non-rigid targets. In the present article, the TTM will be further implemented to explore novel and maybe useful phenomena of ABBs in the forward scattering for rigid complicated targets. On the one hand, the present investigation could be expected to enrich the analysis of scattering of ABBs from not only spherical but also aspherical shapes. On the other hand, it may also provide an alternative for the experimental methods when dealing with forward scattering since the forward scattering signals are easily overwhelmed by either the strong direct signals by source or the strong interference between them. It could be therefore anticipated that the TMM will provide a competitive tool to gain insight into the novel scattering properties and also help to interpret the scattering mechanisms of ABBs in the forward half-space.

The frame of this article is outlined as follows. In Section 2, theoretical formulations of the T-matrix method for the acoustic scattering by rigid obstacles with arbitrary shapes immersed in fluid are presented briefly, including the derivation of the expansion coefficients of the incident ABBs (Section 2.1), a brief frame of TTM for rigid obstacle (Section 2.2), several numerical methods for the Q matrix (Section 2.3) and understandings of form function and target strength (Section 2.4). Subsequently in Section 3.1, convergence study and stability of Q matrix are discussed. Section 3.2–3.5 gives several numerical experiments with particular emphasis on scattering in the forward half-space. Finally, some useful concluding remarks are summarized in Section 4.

2. Theoretical formulation

In the formulation of standard T-matrix method (TMM), all field quantities (including the incident fields, scattered fields, Green's function and unknown surface fields) are expanded in terms of a set of spherical functions in order to obtain the expected transition matrix (T matrix). Specifically, the transition matrix gives a linear relationship between the known coefficients of expansion of the incident wave to the unknown expansion coefficients of the scattered field. It is well known that the incident coefficients of expansion are easily obtained when given a specific scalar basis function for plane wave case. Once the transition matrix is constructed for an object of interest, the scattered fields could be calculated immediately. Likewise, if the acoustic Bessel beams (ABBs), which may be regarded as a superposition of plane waves [9], can be expanded based on an appropriate scalar basis function, the TTM is therefore anticipated to perform very well to solve scattering problems interacted with Bessel beams. So in the following, we will first derive the expansion coefficients of the incident Bessel beam, and then give a brief formula system of TTM for rigid object with arbitrary shapes.

2.1. Expansion coefficients of the incident Bessel beam

Considering an idealized monochromatic zeroth-order Bessel beam (See Fig. 1), which is actually an axisymmetric solution of the Helmholtz equation, the expression of its complex velocity potential could be given [50]

$$\phi_B^i(z, \rho) = \phi_0 \exp(i\kappa z) J_0(\mu\rho) \tag{1}$$

where ϕ_0 is the beam amplitude and i is the unit imaginary number, z and $\rho = \sqrt{x^2 + y^2}$ specify the axial and radial coordinates, $\kappa = k\cos\beta$ and $\mu = k\sin\beta$, satisfying the relation $\kappa^2 + \mu^2 = k^2$, represent the axial and radial wavenumbers, and J_0 is a zeroth-order cylindrical Bessel function of the first kind. Here in Eq. (1), the complex time factor of the form $\exp(-i\omega t)$ has been separated from the spatial dependence of the complex functions. It should be further noted that all field quantities will also have the same harmonic time dependence due to the incident wave, and hence the time dependence will be factored out throughout for convenience.