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Incident energy transfer equation and its solution by collocation spectral method for one-dimensional radiative heat transfer

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ABSTRACT

The ray-effect is a major discretization error in the approximate solution method for the radiative transfer equation (RTE). To overcome this problem, the incident energy transfer equation (IETE) is proposed. The incident energy, instead of radiation intensity, is obtained by directly solving this new equation. Good numerical properties are found for the incident energy transfer equation. To show the properties of numerical solution, the collocation spectral method (CSM) is employed to solve the incident energy transfer equation. Three test cases are taken into account to verify the performance of the incident energy transfer equation. The result shows that the radiative heat flux obtained based on IETE is much more accurate than that based on RTE, which means that the IETE is very effective in eliminating the impacts of ray-effect on the heat flux. However, on the contrary, the radiative intensity obtained based on IETE is less accurate than that based on RTE due to the ray-effect. So, this equation is more suitable for those radiative heat transfer problems, in which the radiation heat flux and incident energy are needed rather than the radiation intensity.

1. Introduction

Radiative heat transfer in absorbing, emitting, and scattering media is important in many scientific and engineering disciplines, and with the development of computer technology, numerical simulation has gradually become an important and useful technique to study the radiative heat transfer. During the past two decades, lots of efforts have been focused on the numerical simulation of radiative transfer process and most of analyzes are based on solving the radiative transfer equation (RTE). Up to now, many numerical methods have been developed to solve the RTE, such as the discrete ordinates method (DOM) [1], finite volume method (FVM) [2,3], finite element method (FEM) [4,5], spectral element method (SEM) [6], lattice Boltzmann method (LBM) [7], and the spectral method (SM) [8–14]. As compared to the convection diffusion equation, the RTE can be considered as a special kind of convection-dominated Eq. [3] without diffusion terms. The convection-dominated property of an equation may cause non-physical oscillation in the numerical simulations. This type of instability occurs in many numerical methods if no special treatment is taken.

In order to reduce the instability caused by the convection-dominated trait of the RTE, the first order partial differential equation (RTE) was analytically transformed into the second order partial differential equation, which is numerically stable. Currently, two different transformed equations have been proposed. One is the even-parity (EPRTE) formulation of the RTE, which is a second order differential equation of the even parity of radiative intensity, and was initially proposed in the field of neutron transport and has been used for decades [15–17]. Another one is the second order radiative transfer equation (SORTE) [18,19], which is a second order differential equation of radiative intensity itself. However, because the existence of the reciprocal of extinction coefficient in the equation, a singularity problem appears for the proposed second order equation in dealing with inhomogeneous media where some locations have very small or null extinction coefficient both for the EPRTE and the SORTE. To overcome the singularity problem of SORTE, a new form of second order radiative transfer equation (named MSORTE) is proposed by Zhao and Liu [20]. These works show obvious effect in eliminating the instability caused by the convection-dominated trait of RTE.

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Another important factor influencing the accuracy of the numerical results is called "ray-effect", which is due to the discretization of the angular dependence. [3,21–26]. This effect arises from the approximation of a continuously varying angular nature of radiation by considering a specified set of discrete angular directions, and is independent of spatial discretization practices. Usually, when RTE or other transfer equations about intensity were adopted to simulate the radiative heat transfer problem in participating media, all the integral terms in those transfer equations, boundary conditions, incident energy and radiative heat flux are approximated by numerical integrals about the radiative intensity. Once such approximation is made, ray effect is encountered. Lots of efforts have been made to reduce the "ray-effect". Lathrop [22] and Chai et al. [25] discussed the ray effect and false scattering in the DOM. Sakami et al. [27] analyzed the ray effect of the modified DOM. Coelho proposed limitary and nonsymmetrical high-order schemes for the DOM [28], and also analyzed compensation of the ray effect and false scattering in the DOM. Relatively speaking, more attention has been paid to the ray effect and false scatting in the DOM than in other methods [29]. Articles [30–34] discussed the ray effect in the traditional ray-tracing method [30], discrete transfer method (DTM) [31], finite volume method (FVM) [32,33] and the radiation element method by ray emission model REM2 [34], etc., and modification measures were proposed.

A very effective way to eliminate the "ray-effect" is solving the radiative integral transfer equations (RITEs) rather than RTE. Because the angular dependency is completely eliminated, the RITEs, which are derived by integrating the RTE over all solid angles, are often solved to obtain the exact solutions of radiative transfer problems [35]. Since the numerical solution of RITEs often leads to dense matrices which may impose restrictions on the computational memory and the execution time, the RITEs are rarely used in practical engineering problems, especially in multidimensional geometries. However, inspired from RITEs, we can solve the equation by incident energy or radiative heat flux directly, instead of using radiative intensity (RTE, SORTE, etc.), the numerical error caused by "ray-effect" would be reduced or even eliminated. In fact, for a lot of radiation heat transfer problems, the numerical results we really need, are the incident energy and the radiative heat flux, not the radiative intensity.

In the present work, the incident energy transfer equation (IETE) is deduced in Section 2. The formulations of the collocation spectral method (CSM) for one-dimensional IETE are presented in detail in Section 3. In Section 4, three test cases of radiative heat transfer in semitransparent media are taken to verify the performance of the method. Finally the last section gives the conclusions.

2. Formulation of the incident energy transfer equation (IETE)

The incident energy is defined as

$$G = \int_{4\pi} I(\Omega)d\Omega = \int_0^{2\pi}\int_0^{\pi} I(\theta,\varphi)\sin\theta d\theta d\varphi \tag{1}$$

In Eq. (1), the bounds of the integral are known and fixed. So, for a known distribution of intensity $I(\theta, \varphi)$, G is constant. To deduce the incident energy transfer equation, a variable incident energy is needed and should be defined at first. In this work, we define the variable incident energy as:

$$g(\theta,\varphi) = \int_0^{\varphi}\int_0^{\theta} I(\theta',\varphi')\sin\theta' d\theta' d\varphi' \quad \varphi\in[0,2\pi], \theta\in[0,\pi] \tag{2}$$

Obviously, $g(\theta, \varphi)$ means the incident radiation in any size solid angle and$g(\pi, 2\pi)=G$, $g(0, \varphi)=g(\theta, 0)=0$. Let$\mu=\cos\theta$, Eq. (1) can be rewritten as:

$$g(\mu,\varphi) = \int_0^{\varphi}\int_{\mu}^{1} I(\mu',\varphi')d\mu' d\varphi', \quad \varphi\in[0,2\pi], \mu\in[-1,1] \tag{3}$$

Take the derivative of Eq. (3) with respect to φ and μ, we can get the expression of radiation intensity as:

$$I(\mu,\varphi) = -\frac{\partial^2 g(\mu,\varphi)}{\partial\varphi\partial\mu} \tag{4}$$

For one-dimensional problems, Eqs. (3) and (4) can be simplified as follows:

$$g(\mu) = 2\pi\int_{\mu}^{1} I(\mu')d\mu', \mu\in[-1,1] \tag{5}$$

$$I(\mu) = -\frac{1}{2\pi}\frac{\partial g(\mu)}{\partial\mu} \tag{6}$$

Compared with the radiation intensity I, a notable characteristics of the variable incident energy g is that the distribution of g is always continuous whether I is continuous or discontinuous. Taking a one-dimensional problem as an example, it can be easily proved as follow:

For any $\mu\in[-1, 1]$, we have

$$\begin{aligned} g(\mu^+) &= 2\pi\int_{\mu^+}^{1} I(\mu')d\mu' = \lim_{\Delta\mu\to 0}\left(2\pi\int_{\mu+\Delta\mu}^{1} I(\mu')d\mu'\right) \\ &= 2\pi\int_{\mu}^{1} I(\mu')d\mu' - \lim_{\Delta\mu\to 0}\left(2\pi\int_{\mu}^{\mu+\Delta\mu} I(\mu')d\mu'\right) \\ &= g(\mu) - \lim_{\Delta\mu\to 0}(2\pi I(\mu+\Delta\mu)\Delta\mu) \\ &= g(\mu) \end{aligned}$$

and

$$\begin{aligned} g(\mu^-) &= 2\pi\int_{\mu^-}^{1} I(\mu')d\mu' = \lim_{\Delta\mu\to 0}\left(2\pi\int_{\mu-\Delta\mu}^{1} I(\mu')d\mu'\right) \\ &= 2\pi\int_{\mu}^{1} I(\mu')d\mu' + \lim_{\Delta\mu\to 0}\left(2\pi\int_{\mu-\Delta\mu}^{\mu} I(\mu')d\mu'\right) \\ &= g(\mu) + \lim_{\Delta\mu\to 0}(2\pi I(\mu-\Delta\mu)\Delta\mu) \\ &= g(\mu) \end{aligned}$$

Obviously, $g(\mu^+)=g(\mu)=g(\mu^-)$, therefore $g(\mu)$ is continuous and the proof is completed.

The governing equation for radiative heat transfer in absorbing-emitting and scattering medium in term of radiation intensity reads [36]:

$$\begin{aligned} &\Omega\cdot\nabla I(r,\Omega) + (k_a+k_s)I(r,\Omega) \\ &\quad - \frac{k_s}{4\pi}\int_{\Omega'=4\pi}\Phi(\Omega'\to\Omega)I(r,\Omega)d\Omega' = k_a I_b(r) \end{aligned} \tag{7}$$

with boundary conditions

$$I(r,\Omega) = I^S(r,\Omega) + \varepsilon I_b(r) + \frac{\rho}{\pi}\int_{n\cdot\Omega'} I(r,\Omega')d\Omega' \tag{8}$$

where all the symbols' explanations are the same as in [37]

Substituting Eq. (4) into Eq. (7) results in the following IETE:

$$\begin{aligned} &\Omega\cdot\nabla\frac{\partial^2 g(\mu,\varphi)}{\partial\varphi\partial\mu} + (k_a+k_s)\frac{\partial^2 g(\mu,\varphi)}{\partial\varphi\partial\mu} \\ &\quad - \frac{k_s}{4\pi}\int_{\Omega'=4\pi}\Phi(\Omega'\to\Omega)\frac{\partial^2 g(\mu,\varphi)}{\partial\varphi\partial\mu}d\Omega' = -k_a I_b(r) \end{aligned} \tag{9}$$