Journal of Quantitative Spectroscopy & Radiative Transfer 200 (2017) 215–224

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

A goal-based angular adaptivity method for thermal radiation modelling in non grey media

Laurent Soucasse[a,*], Steven Dargaville[a], Andrew G Buchan[a,b], Christopher C Pain[a]

[a] Applied Modelling and Computation Group, Department of Earth Science and Engineering, Imperial College London, SW7 2AZ, United Kingdom
[b] School of Engineering and Material Science, Queen Mary University of London, E1 4NS, United Kingdom

ARTICLE INFO

Article history:
Received 24 March 2017
Revised 12 May 2017
Accepted 9 June 2017
Available online 19 June 2017

Keywords:
Goal-based adaptivity
Angular adaptivity
Coupled flow-radiation
Non grey media

ABSTRACT

This paper investigates for the first time a goal-based angular adaptivity method for thermal radiation transport, suitable for non grey media when the radiation field is coupled with an unsteady flow field through an energy balance. Anisotropic angular adaptivity is achieved by using a Haar wavelet finite element expansion that forms a hierarchical angular basis with compact support and does not require any angular interpolation in space. The novelty of this work lies in (1) the definition of a target functional to compute the goal-based error measure equal to the radiative source term of the energy balance, which is the quantity of interest in the context of coupled flow-radiation calculations; (2) the use of different optimal angular resolutions for each absorption coefficient class, built from a global model of the radiative properties of the medium. The accuracy and efficiency of the goal-based angular adaptivity method is assessed in a coupled flow-radiation problem relevant for air pollution modelling in street canyons. Compared to a uniform Haar wavelet expansion, the adapted resolution uses 5 times fewer angular basis functions and is 6.5 times quicker, given the same accuracy in the radiative source term.

1. Introduction

Numerical approximations to the Radiative Transfer Equation (RTE) remain a computational challenge and an intense research area. The difficulty comes from the dependence of the radiative intensity in space and angle, in time when the radiation field is coupled with an unsteady flow, and in wavenumber as the absorption spectrum of common radiating gases like water vapour or carbon dioxide are made of millions of lines. The emergence of parallel computing has made the use of detailed reference methods for solving the RTE, such as the Monte Carlo method or the ray-tracing method, more and more practical. The coupling of these reference methods with a Direct Numerical Simulation (DNS) of a turbulent flow has been for instance recently achieved in Ref. [1]. However, a strong interest persists in reducing the dimension of the discretised RTE for rapid modelling and engineering purposes as well as for simulating large scale systems out of the scope of reference methods.

A substantial effort has been devoted in the literature to reduce the dimension of the wavenumber space and has lead to the development of correlated-k models, Statistical Narrow Band models or global models. A review of these different approaches is given by Taine and Soufiani [2]. Global models, which consist in reordering the absorption spectrum according to the value of the absorption coefficient over the whole spectral domain, are the most computationally efficient and are favoured for 3D applications, although they suffer from inaccuracies in heterogeneous media.

In the thermal radiation community, the angular and spatial discretisation are most often achieved by combining the Discrete Ordinates Method (DOM or S_N expansion) in angle with the Finite Volume Method (FVM) in space [3,4], although the finite element method also provides a general framework for both the angle and space dimensions [5]. A promising way to reduce the dimension of these discretised space-angle systems is the use of adaptive discretisation methods that only refine the regions of interest. These adaptive algorithms are driven by error estimators and include regular adaptive methods where the solution error is reduced uniformly over the whole phase-space domain and goal-based adaptive methods where the solution error is reduced with respect to a target functional.

Several spatial adaptivity schemes for radiation transport have been developed during the past decade. An Adaptive Mesh Refinement (AMR) technique, based on a hierarchy of structured spatial meshes, was used by Ogando and Velarde [6], combined with a S_N expansion in angle. Ragusa [7] proposed a regular error measure, using the Hessian of the discretised spatial flux, while Lathouwers [8] and Goffin et al. [9] experimented with goal-based error measures with various target functionals. More recently, Yang

* Corresponding author.
E-mail address: l.soucasse@imperial.ac.uk (L. Soucasse).

http://dx.doi.org/10.1016/j.jqsrt.2017.06.015

and Yuan [10] developed a *h*-refinement technique for simple corner balance scheme for application to radiation transport coupled with Lagrangian hydrodynamics. Although all these space adaptivity techniques are successful in reducing the number of spatial degrees of freedom, their efficient implementation in parallel remains challenging.

A first attempt at adapting the angular resolution was proposed by Ackroyd and Wilson [11] who used a variable order spherical harmonic expansion (P_N) across space using an a priori knowledge of the spatial variation of the physical properties of the medium. Using a P_N expansion is advantageous for angular adaptivity because it forms a hierarchical basis and it does not need any angular interpolation in space. Goffin et al. took advantage of this property to develop goal-based angular adaptivity algorithms using a P_N expansion [12]. Alternative hierarchical basis for angular discretisation can be formed using wavelet theory and multi-resolution analysis [13]. Compared to spherical harmonics, wavelets have the advantage of adapting anisotropically due to their compact support. Watson [14] and Goffin et al. [15] both made use of wavelet-based angular adaptivity methods with Haar wavelets and octahedral linear wavelets respectively. Other adaptive schemes have also been developed for non-hierarchical basis, such as the S_N expansion [16]. Kópházy and Lathouwers [17] proposed a generalised framework for local angular refinement with arbitrary angular basis functions and discontinuous Galerkin discretisation in space and angle, but the method involves complex algebra to compute the numerical fluxes. Moreover, alternative techniques exist in order to reduce the angular dimension, such as reduced order models [18,19].

Most of these angular adaptivity methods have been applied for problems in nuclear engineering and have not been applied in problems with thermal radiation. In many engineering or environmental applications, the thermal radiation field is coupled with a flow field through an energy balance. In that case the key quantity to predict is the radiative source term, namely the balance between the radiation absorbed and emitted in the medium. In this paper, we apply angular adaptivity for problems in thermal radiation and derive a goal-based adaptivity method that optimises the angular resolution to accurately compute the radiative source term. In order to perform anisotropic angular adaptivity, we make use of a Haar wavelet angular expansion, which is a hierarchical version of a S_N expansion and is compactly supported. The angular dependence of the radiation field strongly depends on the optical thickness of the medium, and hence we apply different angular resolutions for different absorption coefficient classes built from a global model of the radiative properties of the medium.

The layout of the article is as follows. The numerical methods and models we use to discretise the RTE are described in Section 2 and the goal-based adaptivity algorithm is presented in Section 3. The accuracy and efficiency of our goal-based angular adaptivity method is assessed in Section 4 in a coupled flow-radiation problem relevant for air pollution modelling in street canyons.

2. Radiative tranfer equation and numerical methods

We consider in this work a non scattering medium of optical index equal to 1 at local thermal equilibrium, hence we can write the RTE as

$$\mathbf{\Omega} \cdot \nabla I(\nu, \mathbf{r}, \mathbf{\Omega}) = \kappa(\nu)(I_b(\nu, T(\mathbf{r})) - I(\nu, \mathbf{r}, \mathbf{\Omega})), \quad (1)$$

where $I(\nu, \mathbf{r}, \mathbf{\Omega})$ is the radiative intensity at point $\mathbf{r}$, direction $\mathbf{\Omega}$ and wavenumber ν and $I_b(\nu, T(\mathbf{r}))$ is the blackbody intensity at local temperature $T(\mathbf{r})$. The absorption coefficient $\kappa(\nu)$ is assumed to be homogeneous for the practical case addressed in Section 4, although this is not a restriction of the numerical methods and the adaptivity algorithm presented in this paper. The associated boundary condition for a diffuse opaque wall of grey emissivity ε is

$$I(\nu, \mathbf{r}_w) = \varepsilon(\mathbf{r}_w) I_b(\nu, T(\mathbf{r}_w)) + \frac{1-\varepsilon(\mathbf{r}_w)}{\pi} \int_{\mathbf{\Omega}'\cdot\mathbf{n}<0} I_\nu(\mathbf{r}_w, \mathbf{\Omega}') |\mathbf{\Omega}' \cdot \mathbf{n}| d\mathbf{\Omega}', \quad (2)$$

for points $\mathbf{r}_w$ and directions $\mathbf{\Omega}$, such that $\mathbf{\Omega}\cdot \mathbf{n} > 0$, $\mathbf{n}$ being the wall normal directed towards the inside of the domain.

The radiative source term is equal to the opposite of the divergence of the radiative flux $\mathbf{q}^{rad}$

$$-\nabla \cdot \mathbf{q}^{rad}(\mathbf{r}) = \int_{\mathbf{\Omega}} \int_{\nu} \kappa(\nu)(I(\nu, \mathbf{r}, \mathbf{\Omega}) - I_b(\nu, T(\mathbf{r}))) d\nu d\mathbf{\Omega}. \quad (3)$$

The radiative source term goes into the energy balance of the material system: it represents the difference between the absorption and emission of radiative energy by the medium. It is this key quantity that we will try to optimise with the adaptive algorithm.

2.1. Radiative property modelling

In order to model the spectral dependence of the absorption coefficient, we make use of a global model based on an absorption distribution function [20] defined by

$$\mathcal{F}(k) = \frac{\pi}{\sigma T_{\mathrm{ref}}^4} \int_{\nu,\, \kappa_\nu(T_{\mathrm{ref}}) \le k} I_b(\nu, T_{\mathrm{ref}}) d\nu. \quad (4)$$

This function is discretised in intervals $[k_i^-; k_i^+]$ of averaged value k_i. The weights of this distribution associated with each interval i are defined as $a_i = \mathcal{F}(k_i^+) - \mathcal{F}(k_i^-)$. Eqs. (1) and (2) then become

$$\mathbf{\Omega} \cdot \nabla I_i(\mathbf{r}, \mathbf{\Omega}) = k_i \left(\frac{a_i \sigma T^4(\mathbf{r})}{\pi} - I_i(\mathbf{r}, \mathbf{\Omega}) \right), \quad (5)$$

$$I_i(\mathbf{r}_w) = \varepsilon(\mathbf{r}_w) \frac{a_i \sigma T^4(\mathbf{r}_w)}{\pi} + \frac{1-\varepsilon(\mathbf{r}_w)}{\pi} \int_{\mathbf{\Omega}'\cdot\mathbf{n}<0} I_i(\mathbf{r}_w, \mathbf{\Omega}') |\mathbf{\Omega}' \cdot \mathbf{n}| d\mathbf{\Omega}', \quad (6)$$

where σ is the Stefan–Boltzmann constant. The total intensity integrated over the wavenumber is simply retrieved by summing the contribution of each k-class: $I = \int I(\nu) d\nu = \sum_i I_i$.

This global model is advantageous because the integration over the wavenumber is replaced by an integration over the absoption coefficient k, for which a coarse discretisation is sufficient. Another benefit is that we will be able to associate to each k-class a different angular resolution, as the angular dependence of the radiative intensity is known to be affected by the optical thickness of the medium. As a homogeneous medium is considered, the derivation of Eqs. (5) and (6) is exact and the model error is associated with the numerical discretisation in k. However, additional assumptions are required to extend the model to heterogeneous media (see for instance Refs. [21–23]). The model parameters for the medium considered in Section 4 are given in Appendix A.

2.2. Space-angle finite element discretisation

A Sub-Grid Scale (SGS) model (or variational multiscale method, see Ref. [24]) is used to discretised the RTE in space. The full solution $I_i(\mathbf{r}, \mathbf{\Omega})$ is decomposed into a coarse component $\bar{I}_i(\mathbf{r}, \mathbf{\Omega})$ and a sub-grid component $\tilde{I}_i(\mathbf{r}, \mathbf{\Omega})$, both approximated by a finite element formulation. The coarse component lies in a continuous finite element space, spanned by η_N basis functions, and the sub-grid component lies in a discontinuous space, spanned by η_Q basis functions, as follow

$$I_i(\mathbf{r}, \mathbf{\Omega}) \simeq \sum_{j=1}^{\eta_N} N_j(\mathbf{r}) \bar{I}_{ij}(\mathbf{\Omega}) + \sum_{j=1}^{\eta_Q} Q_j(\mathbf{r}) \tilde{I}_{ij}(\mathbf{\Omega}). \quad (7)$$