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Backward and forward Monte Carlo method for vector radiative transfer in a two-dimensional graded index medium

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ABSTRACT

In vector radiative transfer, backward ray tracing is seldom used. We present a backward and forward Monte Carlo method to simulate vector radiative transfer in a two-dimensional graded index medium, which is new and different from the conventional Monte Carlo method. The backward and forward Monte Carlo method involves dividing the ray tracing into two processes backward tracing and forward tracing. In multidimensional graded index media, the trajectory of a ray is usually a three-dimensional curve. During the transport of a polarization ellipse, the curved ray trajectory will induce geometrical effects and cause Stokes parameters to continuously change. The solution processes for a non-scattering medium and an anisotropic scattering medium are analysed. We also analyse some parameters that influence the Stokes vector in two-dimensional graded index media. The research shows that the Q component of the Stokes vector cannot be ignored. However, the U and V components of the Stokes vector are very small.

1. Introduction

In recent decades, the vector radiative transfer theory has been successfully applied to thermal radiation in passive and active remote sensing [1–3]. In most studies, the model of the scattering medium is homogeneous and one-dimensional. However, many problems are non-uniform and multidimensional, such as the remote sensing of the Earth, the radiation of an atmospheric cloud, and the heat transfer in a high temperature furnace [4–6]. To study the spatial distributions of radiation in an inhomogeneous refractive index scattering medium, it is necessary to develop an analytical theory and numerical solutions to two-dimensional or three-dimensional radiative transfer.

Solution methods for the vector radiative transfer equation (VRTE) in graded index (GRIN) media have long been under development. Lau and Watson studied the radiative transfer equation for polarized light transport in GRIN media [7]. Zhao et al. [8,9] presented a complete derivation of the VRTE for the GRIN vector radiative transfer equation (GVRTE). Ben et al. used the Monte Carlo (MC) method to simulate vector radiative transfer in one-dimensional GRIN media [10].

In two-dimensional GRIN media, ray transport follows a curved trajectory determined by Fermat's principle. The trajectory of the ray in the medium is usually a three-dimensional curve. Considering the polarization of the ray beam, the curved ray trajectory will induce geometrical effects on the transfer of the polarization ellipse and lead to the Stokes parameters continuously changing during transport. The solutions to vector radiative transfer in multidimensional GRIN media are more complicated than those in a uniform index medium. To the best of our knowledge, there is no study regarding the VRTE in multidimensional GRIN media.

Forward ray tracing follows rays from a light source to an object. In spite of the fact that forward ray tracing can be used to accurately determine each object, it remains highly inefficient. Consider, for example, passive target detection using the polarized components of infrared signatures [11,12]. Because many rays from the light source never pass through the detector in this situation, for efficiency, the backward ray tracing method is introduced [13–15]. However, the conventional scalar backward Monte Carlo (BMC) method is not applicable in solving vector radiative transfer because reflection, refraction and scattering events cannot be treated in reverse order.

A backward and forward Monte Carlo (BFMC) method has been developed for the problem of vector radiative transfer in one-dimensional GRIN media [16]. The BFMC method involves dividing the ray tracing into two processes backward tracing and forward tracing. The change in the Stokes vector in vector radiative transfer caused by moving, scattering, refraction and reflection has been solved using the BFMC method. In this paper, the BFMC method is developed to solve a two-dimensional problem. We analyse some

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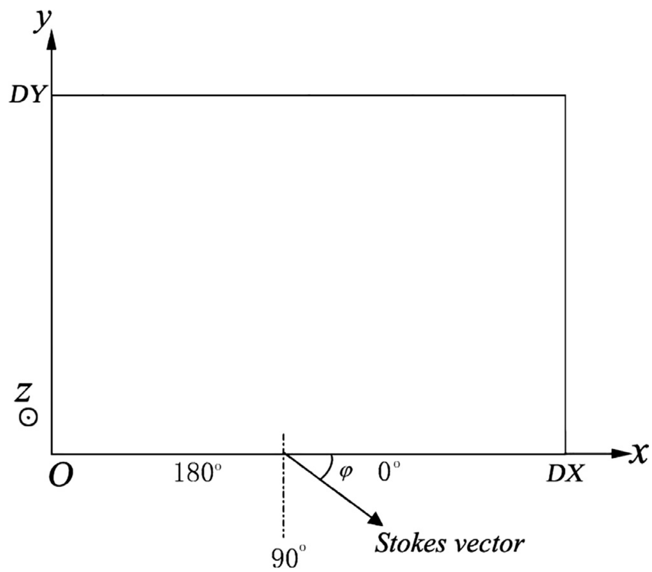

Fig. 1. Two-dimensional GRIN medium with a cross-sectional area of $DX \times DY(\mathrm{m}^2)$ and $DX = DY = 0.1\mathrm{m}$. θ is the angle between the ray direction and the z-axis, and φ is the azimuthal angle.

parameters influencing the Stokes vector in two-dimensional GRIN media. In contrast to the conventional MC method, the BFMC method does not involve angular discretization. Therefore, highly accurate results can be achieved.

2. Physical model and solution method

2.1. Physical model

In this paper, we study the Stokes vector at the bottom of a two-dimensional GRIN medium. Fig. 1 shows a two-dimensional GRIN medium with a cross-sectional area of $DX \times DY(\mathrm{m}^2)$, where $DX = DY = 0.1\mathrm{m}$. The medium is assumed to be an isothermal grey medium with an extinction coefficient κ_e and scattering albedo ω. The refractive distribution of the medium is $n(x,y)$. The surface between the medium and the surroundings is assumed to be semi-transparent and specularly reflective, obeying Snell's law and Fresnel's law.

At present, many scholars have paid more attention to the apparent emissivities of this model. The scalar radiative transfer of two-dimensional GRIN media has been studied by using the curved ray tracing (CRT) method [17–19] and the finite volume method [20]. In two-dimensional problems, the curved ray tracing technology is effective for solving the problem of emissivity.

2.2. Stokes vector

In the study of vector radiative transfer, a beam carries information not only in regard to energy but also polarization. In this paper, the Stokes vector $\mathbf{S} = (I, Q, U, V)^T$ [21] is used to describe the intensity and polarized states of a ray. I is the intensity, Q is the linear polarization aligned parallel or perpendicular to the z-axis, U is the linear polarization aligned $\pm 45^\circ$ with respect to the z-axis, and V is the circular polarization. In the Stokes vector, the first component (I) denotes the energy carried by a photon, while the other components (Q, U, V) indicate the states of polarization. To ensure the conservation of energy in the MC simulation, normalization must be applied to the Stokes vector regarding the propagation of a photon. The I Stokes element of the photon must always be equal to one as it propagates along its trajectory. The normalized Stokes vector is written as [22]

$$\mathbf{S}' = \frac{\mathbf{S}}{I} = \begin{pmatrix} 1 \\ Q/I \\ U/I \\ V/I \end{pmatrix}. \tag{1}$$

The propagation process of light in a medium is accompanied by the physical processes of moving, refraction, reflection, scattering, and absorption. The following describes the changes in the Stokes vector during these processes.

2.2.1. Ray propagating through a gradient-index medium

Absorption, reflection and scattering are not initially considered in the sub process of light propagating through a medium. The derived GVRTE for a non-participating medium is as follows [7,8]

$$n^2 \frac{\mathrm{d}}{\mathrm{d}s}\left[\frac{\mathbf{S}}{n^2}\right] + \mathbf{P}_\kappa \mathbf{S} = 0, \tag{2}$$

where s is the coordinate along the ray trajectory and n is the refractive index. The matrix $\mathbf{P}_\kappa$ is not a physical property of the medium; it is a matrix accounting for the rotation of the polarization ellipse, determined based on Rytov's law, and is defined as [23]

$$\mathbf{P}_\kappa = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2\kappa & 0 \\ 0 & -2\kappa & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3}$$

where κ is the torsion of the ray trajectory. The torsion κ along the trajectory should also be determined based on the solution of the transport equation, and it changes continuously. The ray's trajectory is a plane in a one-dimensional non-scattering GRIN medium; thus, κ is zero. Therefore, in the process of ray propagation through a one-dimensional non-scattering GRIN medium, the Stokes vector can be considered invariant after the normalization. However, in two-dimensional GRIN media, the trajectory of the ray is usually a three-dimensional space curve, and κ is not equal to zero. Eq. (2) shows that it is necessary to consider the change in the Stokes vector during the process of a ray moving through two-dimensional GRIN media.

The torsion κ is not easy to obtain; hence, we adopt an approximate strategy to calculate the Stokes vector of a ray during the moving process. A similar multilayer approximation strategy has been used to trace rays and establish virtual sub layers in a one-dimensional GRIN medium [10]. In this paper, the Runge–Kutta ray tracing (RKRT) method is used to trace rays and establish virtual sub layers.

The RKRT method is able to accurately track the trajectory of a ray [18,24]. As illustrated in Fig. 2, the black line is the backward ray tracing, the grey line is the forward ray tracing, and the Stokes vector is calculated during the forward ray tracing process. To show them more clearly, we do not overlap the ray tracing lines. We can obtain the spatial coordinates and directions of the rays at each iteration point.

To improve our understanding, we illustrate the forward ray tracing process (grey line in Fig. 2) in Fig. 3. Then, the virtual sub layers are established using the iteration points of the RKRT method, as shown in Fig. 3. The dotted lines at the iteration points of the RKRT method represent the virtual sub layers. When the step-size of the RKRT method is small enough, refraction is used to approximately simulate the continuous variety of the Stokes vector during the process of a ray propagating through two-dimensional GRIN media.