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Electromagnetic scattering and emission by a fixed multi-particle object in local thermal equilibrium: General formalism

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ABSTRACT

The majority of previous studies of the interaction of individual particles and multi-particle groups with electromagnetic field have focused on either elastic scattering in the presence of an external field or self-emission of electromagnetic radiation. In this paper we apply semi-classical fluctuational electrodynamics to address the ubiquitous scenario wherein a fixed particle or a fixed multi-particle group is exposed to an external quasi-polychromatic electromagnetic field as well as thermally emits its own electromagnetic radiation. We summarize the main relevant axioms of fluctuational electrodynamics, formulate in maximally rigorous mathematical terms the general scattering–emission problem for a fixed object, and derive such fundamental corollaries as the scattering–emission volume integral equation, the Lippmann–Schwinger equation for the dyadic transition operator, the multi-particle scattering–emission equations, and the far-field limit. We show that in the framework of fluctuational electrodynamics, the computation of the self-emitted component of the total field is completely separated from that of the elastically scattered field. The same is true of the computation of the emitted and elastically scattered components of quadratic/bilinear forms in the total electromagnetic field. These results pave the way to the practical computation of relevant optical observables.

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1. Introduction

The standard treatment of the scattering of electromagnetic waves by particles and multi-particle groups [1–20] has traditionally been based on "deterministic" macroscopic electromagnetics [21–24] and as such has not included explicitly the "stochastic" phenomenon of thermal emission by bodies having non-zero absolute temperatures. In contrast, a large body of recent publications (see, e.g., Refs. [25–39] and references therein) have focused on the study of (near-field) energy transfer in thermally emitting physical systems (including many-particle groups) using the semi-classical "fluctuational" electrodynamics (FED) [40–44]. However, there are practical situations wherein thermal emission processes are accompanied by elastic scattering of external electromagnetic radiation [36]. An important example of such mixed scenario is a cloud of particles in a planetary atmosphere which can both scatter the incident stellar light at near-infrared wavelengths as well as emit its own near-infrared radiation (see, e.g., Refs. [45–49] and references therein). Fortunately, by its very construct, FED is ideally suited to address such situations.

Indeed, FED amounts to a reformulation of the macroscopic Maxwell equations (MMEs) wherein the usual "deterministic" volume charge density is supplemented by the volume density of the "stochastic" thermal electric current. The latter is caused by randomly fluctuating positions of elementary charges constituting a body at a non-zero absolute temperature. As a consequence, the modified MMEs describe simultaneously the processes of thermal emission as well as elastic scattering, albeit at the price of added mathematical complexity.

The classical formalism based on the MMEs and developed for the study of elastic electromagnetic scattering by single- and multi-particle objects is well developed [1–10,12–20]. The next obvious step is to analyze in a systematic way how the main aspects of this elastic-scattering formalism are modified by the inclusion of thermal emission effects. This paper is intended to facilitate this analysis by summarizing the main relevant axioms of FED, formulating in maximally rigorous mathematical terms the general scattering–emission problem for a fixed (multi-particle) object exposed to a quasi-polychromatic external field, and generalizing such fundamental corollaries of the MMEs as the volume integral equation, the Lippmann–Schwinger equation for the dyadic transition operator, the Foldy equations, and the far-zone approximation. Fundamentally, we show that the FED framework allows one to split the problem of finding the total electromagnetic field into the computation of the self-emitted field and the calculation of the elastically scattered field. Furthermore, we demonstrate that the same is true of the problem of computing second moments of the

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total electromagnetic field. These results are expected to pave the way to the calculation of optical observables encountered in actual practical applications.

2. Stochastic macroscopic Maxwell equations, constitutive relations, and boundary conditions

Under the assumption that all media involved are nonmagnetic, the system of four stochastic MMEs for the instantaneous macroscopic electromagnetic field at an arbitrary observation point $\mathbf{r}$ is as follows [44]:

$$\nabla \cdot \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t), \tag{1}$$

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\mu_0 \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t}, \tag{2}$$

$$\nabla \cdot \mathbf{H}(\mathbf{r}, t) = 0, \tag{3}$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \mathbf{J}(\mathbf{r}, t) + \mathbf{J}^{\mathrm{f}}(\mathbf{r}, t) + \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t}, \tag{4}$$

where we use the SI units, $\mathbf{E}(\mathbf{r},t)$ is the electric and $\mathbf{H}(\mathbf{r},t)$ the magnetic field, $\mathbf{D}(\mathbf{r},t)$ is the electric displacement, $\rho(\mathbf{r},t)$ and $\mathbf{J}(\mathbf{r},t)$ are the macroscopic (free) volume charge density and current density, respectively, $\mathbf{J}^{\mathrm{f}}(\mathbf{r},t)$ is the volume density of the *fluctuating* electric current, and μ_0 is the magnetic permeability of a vacuum. All quantities entering Eqs. (1)–(4) are real-valued functions of time t as well as of spatial coordinates. Implicit in the stochastic MMEs is the continuity equation

$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} + \nabla \cdot \mathbf{J}(\mathbf{r}, t) + \nabla \cdot \mathbf{J}^{\mathrm{f}}(\mathbf{r}, t) = 0, \tag{5}$$

which is obtained by combining the time derivative of Eq. (1) with the divergence of Eq. (4) and making use of the vector identity

$$\nabla \cdot (\nabla \times \mathbf{a}) \equiv 0. \tag{6}$$

Typically Eqs. (1)–(4) must be supplemented by appropriate constitutive relations. In the case of a time-dispersive medium, we have

$$\mathbf{D}(\mathbf{r}, t) = \int_{-\infty}^{t} \mathrm{d}t' \, \varepsilon\left(\mathbf{r}, t - t'\right)\mathbf{E}\left(\mathbf{r}, t'\right), \tag{7}$$

$$\mathbf{J}(\mathbf{r}, t) = \int_{-\infty}^{t} \mathrm{d}t' \, \sigma(\mathbf{r}, t - t')\mathbf{E}(\mathbf{r}, t'), \tag{8}$$

where ε is the electric permittivity and σ is the electric conductivity.

If two different continuous media with finite conductivity are separated by an interface S then it is postulated that the tangential components of the electric and magnetic field vectors are continuous across S:

$$\hat{\mathbf{n}} \times [\mathbf{E}_1(\mathbf{r}, t) - \mathbf{E}_2(\mathbf{r}, t)] \equiv \mathbf{0}, \tag{9}$$

$$\hat{\mathbf{n}} \times [\mathbf{H}_1(\mathbf{r}, t) - \mathbf{H}_2(\mathbf{r}, t)] \equiv \mathbf{0}, \tag{10}$$

where $\mathbf{0}$ is a zero vector and $\hat{\mathbf{n}}$ is a unit vector along the local normal to S.

3. The Poynting theorem

The system of axioms (1)–(4) and (7)–(10) of fluctuational electromagnetics must provide a link to other physical quantities, including those directly measurable with suitable instrumentation. This is accomplished in part by using the Lorentz force postulate which states that if a differential volume element $\mathrm{d}V$ contains a total charge $\rho(\mathbf{r},t)\mathrm{d}V$ moving at a velocity $\mathbf{v}(\mathbf{r},t)$ then the force exerted by the electromagnetic field on that charge is

$$\mathrm{d}\mathbf{F} = \rho(\mathbf{r}, t)\mathbf{E}(\mathbf{r}, t)\mathrm{d}V + \mu_0 \rho(\mathbf{r}, t)\mathbf{v}(\mathbf{r}, t) \times \mathbf{H}(\mathbf{r}, t)\mathrm{d}V. \tag{11}$$

Upon scalar multiplying $\mathrm{d}\mathbf{F}$ by $\mathbf{v}(\mathbf{r},t)$, we see that the magnetic field does no work, while for the local charge $\rho(\mathbf{r},t)\mathrm{d}V$ the rate of doing work by the electric field is $\rho(\mathbf{r},t)\mathbf{v}(\mathbf{r},t) \cdot \mathbf{E}(\mathbf{r},t)\mathrm{d}V$. Thus the total rate of work done by the electromagnetic field inside a finite volume V is given by

$$Q = \int_V \mathrm{d}^3\mathbf{r} \, [\mathbf{J}(\mathbf{r}, t) + \mathbf{J}^{\mathrm{f}}(\mathbf{r}, t)] \cdot \mathbf{E}(\mathbf{r}, t). \tag{12}$$

We now make use of Eqs. (2) and (4), the vector identity

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b}) \tag{13}$$

with $\mathbf{a} = \mathbf{E}$ and $\mathbf{b} = \mathbf{H}$, and the Gauss theorem

$$\int_V \mathrm{d}^3\mathbf{r} \, \nabla \cdot \mathbf{A}(\mathbf{r}) = \int_S \mathrm{d}^2\mathbf{r} \, \mathbf{A}(\mathbf{r}) \cdot \hat{\mathbf{n}}(\mathbf{r}), \tag{14}$$

where S is the closed surface bounding V and $\hat{\mathbf{n}}(\mathbf{r})$ is a unit vector in the direction of the local outward normal to S. The result is the so-called Poynting theorem quantifying the energy budget of the volume V:

$$-\int_S \mathrm{d}^2\mathbf{r} \, \mathbf{S}(\mathbf{r}, t) \cdot \hat{\mathbf{n}}(\mathbf{r}) = Q + \frac{\mathrm{d}U}{\mathrm{d}t}, \tag{15}$$

where

$$\mathbf{S}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{H}(\mathbf{r}, t) \tag{16}$$

is the Poynting vector and the term

$$\frac{\mathrm{d}U}{\mathrm{d}t} = \int_V \mathrm{d}^3\mathbf{r} \left[\mathbf{E}(\mathbf{r}, t) \cdot \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t} + \mu_0 \mathbf{H}(\mathbf{r}, t) \cdot \frac{\partial \mathbf{H}(\mathbf{r}, t)}{\partial t}\right] \tag{17}$$

accounts for both the rate of change of the stored electromagnetic energy in V and the rate of energy dissipated by the material in V [24]. It is postulated that the left-hand side of Eq. (15) represents the net flow of electromagnetic energy entering V.

4. Fourier decomposition

Let us express all time-varying fields entering the stochastic MMEs in terms of time-harmonic components using the Fourier analysis:

$$\mathbf{E}(\mathbf{r}, t) = \int_{-\infty}^{\infty} \mathrm{d}\omega \, \mathbf{E}(\mathbf{r}, \omega) \exp(-\mathrm{i}\omega t) \tag{18}$$

and similarly for $\mathbf{H}(\mathbf{r},t)$, $\mathbf{D}(\mathbf{r},t)$, $\rho(\mathbf{r},t)$, $\mathbf{J}(\mathbf{r},t)$, and $\mathbf{J}^{\mathrm{f}}(\mathbf{r},t)$, where $\mathrm{i} = (-1)^{1/2}$. The respective frequency spectra are given by the Fourier transforms

$$\mathbf{E}(\mathbf{r}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}t \, \mathbf{E}(\mathbf{r}, t) \exp(\mathrm{i}\omega t), \text{ etc.} \tag{19}$$

It is straightforward to verify that since the actual physical fields are real-valued, the frequency spectra satisfy the symmetry relations

$$\mathbf{E}(\mathbf{r}, -\omega) = [\mathbf{E}(\mathbf{r}, \omega)]^*, \text{ etc.,} \tag{20}$$

where the asterisk denotes the complex-conjugate value.

By virtue of the Fourier integral theorem, the frequency-domain system of the stochastic Maxwell equations and boundary conditions takes the form

$$\nabla \cdot [\varepsilon(\mathbf{r}, \omega)\mathbf{E}(\mathbf{r}, \omega)] = -\frac{\mathrm{i}}{\omega} \nabla \cdot \mathbf{J}^{\mathrm{f}}(\mathbf{r}, \omega), \tag{21}$$

$$\nabla \times \mathbf{E}(\mathbf{r}, \omega) = \mathrm{i}\omega\mu_0 \mathbf{H}(\mathbf{r}, \omega), \tag{22}$$

$$\nabla \cdot \mathbf{H}(\mathbf{r}, \omega) = 0, \tag{23}$$