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Short Communication

Electromagnetic scattering by spheroidal volumes of discrete random medium

Michael I. Mishchenko[a,*], Janna M. Dlugach[b]

[a] NASA Goddard Institute for Space Studies, 2880 Broadway, New York, NY 10025, USA
[b] Main Astronomical Observatory of the National Academy of Sciences of Ukraine, 27 Zabolotny Str., 03680, Kyiv, Ukraine

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ABSTRACT

We use the superposition *T*-matrix method to compare the far-field scattering matrices generated by spheroidal and spherical volumes of discrete random medium having the same volume and populated by identical spherical particles. Our results fully confirm the robustness of the previously identified coherent and diffuse scattering regimes and associated optical phenomena exhibited by spherical particulate volumes and support their explanation in terms of the interference phenomenon coupled with the order-of-scattering expansion of the far-field Foldy equations. We also show that increasing nonsphericity of particulate volumes causes discernible (albeit less pronounced) optical effects in forward and backscattering directions and explain them in terms of the same interference/multiple-scattering phenomenon.

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1. Introduction

The use of direct, numerically exact computer solvers of the macroscopic Maxwell equations to study electromagnetic scattering by volumes of discrete random medium (DRM) has been a hot topic over the past decade (see, e.g., Refs. [1–6] and the comprehensive reference list in the recent review [7]). In particular, the effects of domain size [8], particle size [9], particle refractive index [1,9] (including the imaginary part [10]), and particle packing density [11,12] have been studied in substantial detail. In many publications the statistical randomness of particle positions has been modeled by first running a random-number generator to assign coordinates of N particles quasi-randomly filling a spherical volume of DRM and then averaging over the uniform orientation distribution of the resulting multi-particle configuration (e.g., [1,7] and references therein). Several numerical tests have shown that this approach yields highly repeatable far-field scattering patterns irrespectively of the initial quasi-random set of particle positions within the volume, in a stark contrast to the patterns caused by fully ordered multi-particle configurations in random orientation [13]. Furthermore, the use of the analytical orientation-averaging procedure afforded by the superposition *T*-matrix method [14,15] completely eliminates residual statistical "noise" in the angular scattering patterns caused by brute-force numerical ensemble averaging (see, e.g., Refs. [16,17]). The resulting numerical data have been used to study the suppression of the speckle pattern upon ensemble averaging as well as to identify definitively the coherent forward-scattering, diffuse radiative-transfer, and coherent backscattering regimes and their strong dependence on particle characteristics [1,7,18].

The majority of the results thus obtained have relied on the superposition *T*-matrix method and (with the exception of a cube [16] and a cylindrical slab [19] in fixed orientation) on the simplest model of a DRM in the form of a (statistically) spherical particulate volume in random orientation (e.g., [1,7,18]). Importantly, however, this technique is not restricted to spherical volumes of DRM and can be applied effectively to, e.g., spheroidal volumes. This makes it possible to analyze whether the main conclusions of the previous studies reviewed in Ref. [7] remain intact in the case of randomly oriented nonspherical volumes of DRM and whether nonsphericity leads to new discernable effects.

The main objective of this Short Communication is to perform such an analysis. In the following section we briefly introduce the requisite terminology and notation and describe the modeling methodology used in our analysis. The final section contains a discussion and summary of our findings.

2. Modeling methodology

Our analysis parallels that in Refs. [1,7,18] and is based on the comparison of far-field scattering matrices generated by spheroidal as well as spherical volumes of DRM having the same volume V and populated by identical spherical particles. In all computations,

* Corresponding author.
E-mail address: michael.i.mishchenko@nasa.gov (M.I. Mishchenko).

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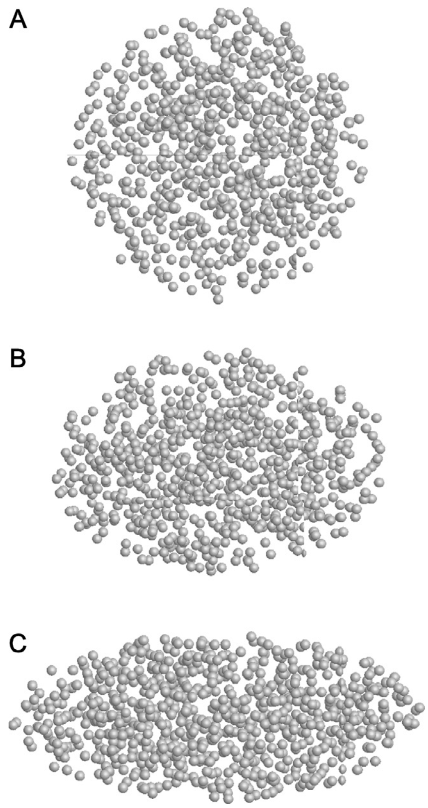

Fig. 1. *V*-equivalent spherical (panel A) and oblate spheroidal (panels B and C) volumes of discrete random medium populated quasi-randomly by $N = 800$ identical spherical particles. The aspect ratio E is 1.5 in panel B and 2.5 in panel C. The volumes are viewed perpendicularly to their short axes.

the particle radius r is fixed at a value implying the particle size parameter $kr = 2$, where k is the wave number in the host medium, while the refractive index of the particles is fixed at $m = 1.31$. These specific values are expected to help identify the presence of the polarization opposition effect [8] observed both in the laboratory [20,21] and in telescopic observations for a class of high-albedo solar system bodies [22,23]. The radius of the spherical volumes R is fixed at a value implying the volume size parameter $kR = 60$. The shape of a prolate or oblate spheroid is defined by its aspect ratio E, i.e., the ratio of the longest to the shortest spheroidal axes, while the lengths of the axes are defined by the requirement that the spheroid have the same volume V.

The initial particle coordinates inside a spherical or spheroidal volume are assigned by running a random-number generator developed by D. W. Mackowski (personal communication; see Ref. [24]) and making sure that the particles do not overlap (see examples of multi-sphere configurations representing spherical and spheroidal volumes of DRM in Fig. 1). This is followed by the summation of the light-scattering results obtained by averaging over the equiprobable orientation distribution of the particulate volume and of its mirror counterpart. It is well known [25,26] that the outcome of this procedure is the symmetric block-diagonal normalized scattering matrix given by

$$\tilde{\mathbf{F}}(\Theta) = \begin{bmatrix} \tilde{F}_{11}(\Theta) & \tilde{F}_{12}(\Theta) & 0 & 0 \\ \tilde{F}_{12}(\Theta) & \tilde{F}_{22}(\Theta) & 0 & 0 \\ 0 & 0 & \tilde{F}_{33}(\Theta) & \tilde{F}_{34}(\Theta) \\ 0 & 0 & -\tilde{F}_{34}(\Theta) & \tilde{F}_{44}(\Theta) \end{bmatrix} \quad (1)$$

with

$$\tilde{F}_{12}(0) = \tilde{F}_{34}(0) = \tilde{F}_{12}(\pi) = \tilde{F}_{34}(\pi) = 0, \quad (2)$$

where $\Theta \in [0, \pi]$ is the scattering angle (i.e., the angle between the incidence and scattering directions) and the (1,1) element (i.e., the phase function) satisfies the standard normalization condition

$$\frac{1}{2}\int_0^{\pi} d\Theta \, \tilde{F}_{11}(\Theta) \, \sin\Theta = 1. \quad (3)$$

Note that the scattering matrix of Eq. (1) is that expected as the asymptotic result of ensemble averaging over an infinite number of quasi-random realizations of a particulate volume. We have demonstrated previously that averaging over all orientations of a single quasi-random realization already yields results representative of those obtained by ensemble averaging. Yet the off-block-diagonal elements, while being much smaller than the block-diagonal ones in the absolute-value sense, do not vanish completely. Therefore, the purpose of averaging over the equiprobable orientation distribution of the particulate volume and of its mirror counterpart is to yield the scattering matrix (1) with the off-block-diagonal elements precisely equal to zero. We have verified that doing that with the superposition *T*-matrix method is in fact numerically equivalent to an artificial symmetrization wherein the off-block-diagonal elements computed for a randomly oriented quasi-random multiparticle group are zeroed out without adding the scattering matrix for the mirror counterpart of the group.

All numerical computations have been performed on the distributed-memory computer cluster of the Main Astronomical Observatory of the Ukrainian National Academy of Sciences using the parallelized version of the superposition *T*-matrix method described in Ref. [15].

3. Discussion and conclusions

Fig. 2 displays all six independent elements of the normalized scattering matrix for a spherical volume of DRM with $N = 800$ and those of the *V*- and *N*-equivalent oblate spheroidal volume with $E = 2.5$. It is remarkable that despite a large difference in the shapes of the two volumes (Fig. 1), the corresponding scattering matrices are very similar. Both phase functions reveal the following common traits:

- a strong coherent forward-scattering effect in the form of almost identical sharp "diffraction" peaks centered at $\Theta = 0$;
- a very smooth and featureless diffuse background at scattering angles extending from 20° to 170°; and
- a coherent backscattering peak centered at $\Theta = 180°$.

All three traits are discussed in Refs. [1,7] and explained in the framework of the order-of-scattering expansion of the far-field Foldy equations. Furthermore, the degree of linear polarization for unpolarized incident light (i.e., the ratio $-\tilde{F}_{12}(\Theta)\tilde{F}_{11}(\Theta)$) exhibits the coherent polarization opposition effect in the form of a narrow minimum at backscattering angles [27]. The qualitative explanation of this phenomenon is given in Refs. [8,28].

Consistent with their interference nature, all manifestations of coherent backscattering intensify with increasing N. This is illustrated in Fig. 3 displaying the ratio $-\tilde{F}_{12}(\Theta)/\tilde{F}_{11}(\Theta)$ and the quantity $[\tilde{F}_{11}(\Theta) - \tilde{F}_{22}(\Theta)]/2$. The latter describes the angular distribution of the cross-polarized scattered intensity in the case of linearly polarized incident light. It is seen indeed that the depth of the backscattering polarization minimum doubles as N increases from 400 to 800, while the height of the backscattering peak in $[\tilde{F}_{11}(\Theta) - \tilde{F}_{22}(\Theta)]/2$ grows by a factor of 1.8.

The results of extensive computations for oblate and prolate spheroidal volumes of DRM with $400 \le N \le 800$ and $1 \le E \le 2$ are quite analogous to those in Figs. 2 and 3 and therefore are not shown.

Despite the somewhat surprising similarity of the $E = 1$ and $E = 2.5$ curves in Fig. 2, we can clearly identify two additional ef-