Journal of Quantitative Spectroscopy & Radiative Transfer 195 (2017) 8–17

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

General description of transverse mode Bessel beams and construction of basis Bessel fields

Jia Jie Wang [a,b,*], Thomas Wriedt [b], James A. Lock [c], Yong Chang Jiao [d]

[a] School of Physics and Optoelectronic Engineering, Xidian University, 710071 Xi'an, China
[b] Foundation Institute of Material Science (IWT), Department of Production Engineering, University of Bremen, Badgasteiner Street 3, 28359 Bremen, Germany
[c] Department of Physics, Cleveland State University, Cleveland, OH 44115, USA
[d] School of Electronic Engineering, Xidian University, 710071 Xi'an, China

ARTICLE INFO

Article history:
Received 5 September 2016
Received in revised form
12 October 2016
Accepted 14 October 2016
Available online 20 October 2016

Keywords:
Bessel beam
Hertz vector potential
Angular spectrum representation
Basis Bessel fields
Light scattering

ABSTRACT

Based on an analysis of polarized Bessel beams using the Hertz vector potentials and the angular spectrum representation (ASR), a general description of transverse mode Bessel beams is proposed. As opposed to the cases of linearly and circularly polarized Bessel beams, the magnetic and electric fields of a Bessel beam in a transverse mode are orthogonal to each other. Both sets of fields together form a complete set of basis Bessel fields, in terms of which an arbitrary Bessel beam can be regarded as a linear combination. The completeness of the basis Bessel fields is analyzed from the perspectives of waveguide theory and vector wave functions. Decompositions of linearly polarized, circularly polarized, and circularly symmetric n-order Bessel beams in terms of basis Bessel fields are given. The results presented in this paper provide a fresh perspective on the description of Bessel beams, which are useful in casting insights into the experimental generation of Bessel beams and the interpretation of light scattering-related problems in practice.

1. Introduction

With a continuous development and application of laser sources, investigation of interactions between various structured laser beams and small particles have attracted increasingly more attention in recent years [1–4]. Among the various shaped beams, there is increasing interest in Bessel beams, which were introduced by Durnin and co-workers [5,6] almost three decades ago. Although ideal Bessel beams cannot be generated, high-quality quasi-Bessel beams can be experimentally realized using a range of different means, e.g. an axicon lens [7,8], spatial light modulator (SLM) [9,10], and other methods [11,12]. Applications of Bessel beams cover a wide range of research fields [13–15], e.g. optical trapping and manipulation [16,17], atom optics [18], and imaging [19].

This paper is a continuation and extension of [20], which was concerned with the general description of ideal Bessel beams of arbitrary order derived using different approaches, for exploring the properties of Bessel beams, and clearing up confusions when dealing with different descriptions of Bessel beams in practical applications. Two different procedures were applied to obtain the fields of an n-order Bessel beam, namely the angular spectrum representation (ASR) method which obtains the fields as a superposition of component plane waves [21–24], and the Davis procedure which obtains the fields from a polarized vector potential [25–30]. Based on the derivation of a class of Davis circularly symmetric Bessel beams using the Hertz vector potentials, it was found [20,31] that they have the same general functional dependence on the cylindrical coordinate system variables (ρ, φ) as do the aplanatic Bessel beams generated using ASR. A general mathematical description of circularly symmetric Bessel beams for four typical types of polarization states was proposed in [20], including $(1, 0)$, which is reminiscent of x-polarized, $(0, 1)$, which is reminiscent of y-polarized, $(1, i)$, which is reminiscent of left-circularly polarized, and $(1, -i)$, which is reminiscent of right-circularly polarized. The generalization of the description renders the Davis type Bessel beam and the ASR type Bessel beam as the two simplest cases of an infinite number of possible circularly symmetric Bessel beams, corresponding to different values of the arbitrary function $g(\alpha_0)$, which depends on the optical system being used. This result is important since it bridges the gap between different descriptions of Bessel beams and exhibits a unification of the different methods of derivation.

* Corresponding author at: School of Physics and Optoelectronic Engineering, Xidian University, 710071 Xi'an, China.
E-mail address: wangjiajie@xidian.edu.cn (J.J. Wang).

http://dx.doi.org/10.1016/j.jqsrt.2016.10.010
0022-4073/

In this paper the generalization procedure used in [20,31] is extended to transverse mode Bessel beams, i.e. the transverse magnetic (TM) polarized and the transverse electric (TE) polarized n-order Bessel beams. This set of polarization bases is typical of that encountered in optical fibers or electromagnetic waveguides, where the magnetic (electric) field is perpendicular to the propagation direction axis, and is called the transverse magnetic (electric) mode, or TM (TE) mode for short. Whereas in optical fibers or in electromagnetic waveguides, the fields must satisfy appropriate boundary conditions at the surface, the fields of an ideal Bessel beam are assumed to be valid over all space. For instance, quasi-Bessel beams in transverse modes can be generated in free space by using a Mach–Zehnder interferometer [12], or an axicon lens [7].

The body of this paper is organized as follows. The fields of Davis Bessel beams in a transverse mode are analyzed in Section 2. The fields of Bessel beams in TM and TE modes generated using ASR are studied in Section 3. A general description of Bessel beams in a transverse mode is proposed in Section 4, showing that the Davis Bessel beams of Section 2 and the aplanatic Bessel beams of Section 3 are the two simplest cases of an infinite number of possible n-order Bessel beams in a transverse mode. Section 5 presents an analysis of basis Bessel fields and the decomposition of various types of polarized Bessel beams in terms of them. Conclusions and discussions are stated in Section 6.

2. Davis Bessel beams using Hertz vector potentials

Briefly, we begin with Maxwell's equations for an electromagnetic wave propagating in an isotropic, dielectric, non-magnetic, and linear medium

$$\nabla\times\mathbf{E} = -i\omega\mathbf{B}, \qquad \nabla\times\mathbf{B} = i\omega\varepsilon\mu\mathbf{E},$$
$$\nabla\cdot\varepsilon\mathbf{E} = 0, \qquad \nabla\cdot\mathbf{B} = 0, \tag{1}$$

where the time dependence is $\exp(i\omega t)$, and ε and μ are the permittivity and permeability of the medium, respectively. Using the Lorenz gauge condition, and introducing the Hertz vector potential (see Sec.6.1 of [32]), we have the vector Helmholtz equation

$$\nabla\nabla\cdot\mathbf{\Pi} + k^2\mathbf{\Pi} = 0, \tag{2}$$

and its two types of independent solutions, the electric potential $\mathbf{\Pi}_e$ and the magnetic potential $\mathbf{\Pi}_m$, resulting in two independent sets of waves

E-type waves

$$\mathbf{E}_e = (k^2 + \nabla\nabla\cdot)\mathbf{\Pi}_e, \quad \mathbf{B}_e = i\omega\mu\varepsilon\nabla\times\mathbf{\Pi}_e, \tag{3}$$

H-type waves

$$\mathbf{E}_m = -i\omega\nabla\times\mathbf{\Pi}_m, \quad \mathbf{B}_m = (k^2 + \nabla\nabla\cdot)\mathbf{\Pi}_m. \tag{4}$$

2.1. Linearly and circularly polarized Bessel beam

Considering the problem in a right-hand Cartesian coordinate system, the vector Helmholtz wave equation simplifies to

$$\frac{\partial^2\mathbf{\Pi}}{\partial x^2} + \frac{\partial^2\mathbf{\Pi}}{\partial x^2} + \frac{\partial^2\mathbf{\Pi}}{\partial x^2} + k^2\mathbf{\Pi} = 0. \tag{5}$$

As described in detail in [20], a linearly x-polarized Bessel beam, a linearly y-polarized Bessel beam, a right-circularly polarized, and a left-circularly polarized Bessel beam can be generated by assuming $\mathbf{\Pi}$ takes the form $\mathbf{\Pi}_m = \Pi_m\mathbf{e}_y = J_n(k_t\rho)(-i)^n e^{in\phi}e^{-ik_z z}\mathbf{e}_y$, $\mathbf{\Pi}_m = \Pi_m\mathbf{e}_x = J_n(k_t\rho)(-i)^n e^{in\phi}e^{-ik_z z}\mathbf{e}_x$, $\mathbf{\Pi}_m = J_n(k_t\rho)(-i)^n e^{in\phi}e^{-ik_z z}(\mathbf{e}_x + i\mathbf{e}_y)$, and $\mathbf{\Pi}_m = J_n(k_t\rho)(-i)^n e^{in\phi}e^{-ik_z z}(\mathbf{e}_x - i\mathbf{e}_y)$, respectively. The radial distance and the azimuthal angle in the transverse plane (x, y) are $\rho = \sqrt{x^2+y^2}$ and $\phi = tan^{-1}(y/x)$, respectively. The transverse and longitudinal wave numbers are denoted as $k_t = k\sin\alpha_0$ and $k_z = k\cos\alpha_0$, respectively. The n-order Bessel function of the first kind is $J_n(\cdot)$. The wavenumber is k, and α_0 is the half-cone angle of the Bessel beam which is defined with respect to the direction of wave propagation. Explicit expressions for the electric and magnetic fields of a linearly x-polarized Bessel beam and a linearly y-polarized Bessel beam were presented as Eqs. (7) and (8) and Eqs. (15) and (16) in [20]. Explicit expressions for a right-circularly polarized and a left-circularly polarized n-order Bessel beam are obtained by a superposition of two orthogonal linearly polarized cases, as given in Eq. (18) in [20]. As we can see, as opposed to a propagating factor $\exp(-ikz)$ carried by a plane wave, ideal Bessel beams carry a propagating factor $\exp(-ikz\cos\alpha_0)$. This implies that, in free space, the group velocity of Bessel beams in the positive z-axis direction is $c\cdot\cos\alpha_0$ which is less than that of a plane wave c [33].

It should be noted that the x-polarized and y-polarized Bessel beams are not circularly symmetric, which can be deduced from the expressions for their time-averaged energy densities $\langle w\rangle$ and their time-averaged Poynting vector power densities $\langle\mathbf{S}\rangle$ given in Eqs. (9)–(11) in [20]. Although the time-averaged energy and Poynting vector power densities of the left-circularly polarized and right-circularly polarized Bessel fields are circularly symmetric, their field components $\mathbf{E}$ and $\mathbf{B}$ are not symmetric [26]. Neither linearly polarized nor circularly polarized Bessel fields can be used to describe circularly symmetric polarized Bessel beams or circularly symmetric scalar Bessel beams, whose electric and magnetic field are symmetric. The Davis-type circularly symmetric Bessel fields can be produced by averaging a polarized Bessel beam with its dual fields [34]. The details of this procedure were described in [20], where the resulting circularly symmetric fields with polarization (1,0), which is reminiscent of x-polarization are given by Eqs. (20–25). Fields with polarization (0,1), which is reminiscent of y-polarization are given by Eqs. (32–37). In addition, fields with polarizations $(1, i)$ and $(1, -i)$, which are reminiscent of left-circularly polarization and right-circularly polarized, are given by Eqs. (39–40) and Eqs. (42) and (43), respectively.

2.2. TE and TM polarized Bessel beams

Considering the vector Helmholtz wave equation of Eq. (2) in a right-hand cylindrical coordinate system, one has

$$\frac{\partial^2\mathbf{\Pi}}{\partial z^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\mathbf{\Pi}}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\mathbf{\Pi}}{\partial\phi^2} + k^2\mathbf{\Pi} = 0. \tag{6}$$

If $\mathbf{\Pi}$ is a longitudinal vector potential polarized along z axis with

$$\mathbf{\Pi}_e = \Pi_e\mathbf{e}_z = J_n(k_t\rho)(-i)^n e^{in\phi}e^{-ik_z z}\mathbf{e}_z, \tag{7}$$

we obtain the electric and magnetic fields

$$E_\rho^{TM} = -iE_0k_zk_tJ'_n(k_t\rho)(-i)^n e^{in\phi}e^{-ik_z z}$$
$$E_\phi^{TM} = \frac{n}{\rho}E_0k_zJ_n(k_t\rho)(-i)^n e^{in\phi}e^{-ik_z z}$$
$$E_z^{TM} = E_0k_t^2J_n(k_t\rho)(-i)^n e^{in\phi}e^{-ik_z z}, \tag{8}$$