Journal of Quantitative Spectroscopy & Radiative Transfer 195 (2017) 18–25

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

On the validity of localized approximation for an on-axis zeroth-order Bessel beam

Gérard Gouesbet [a,*], J.A. Lock [b], L.A. Ambrosio [c], J.J. Wang [d]

[a] CORIA-UMR 6614- Normandie Université, CNRS-Université et INSA de Rouen, Campus Universitaire du Madrillet, 76800 Saint-Etienne-du Rouvray, France

[b] Department of Physics, Cleveland State University, Cleveland, OH 44115, USA

[c] Department of Electrical and Computer Engineering, São Carlos School of Engineering, University of São Paulo, 400, Trabalhador São-Carlense Avenue, São Carlos, SP 13566-590, Brazil

[d] School of Physics and Optoelectronic Engineering, Xidian University, Xi'an, Shannxi 710071, China

ARTICLE INFO

Article history:
Received 11 February 2016
Received in revised form
16 June 2016
Accepted 29 June 2016
Available online 9 July 2016

Keywords:
Generalized Lorenz–Mie theories
Beam shape coefficients
Localized approximations
Bessel beams

ABSTRACT

Localized approximation procedures are efficient ways to evaluate beam shape coefficients of laser beams, and are particularly useful when other methods are ineffective or inefficient. Several papers in the literature have reported the use of such procedures to evaluate the beam shape coefficients of Bessel beams. Examining the specific case of an on-axis zeroth-order Bessel beam, we demonstrate that localized approximation procedures are valid only for small axicon angles.

1. Introduction

When describing a laser beam for use in various light scattering theories such as Generalized Lorenz–Mie Theories (GLMTs), e.g. [1] and the Extended Boundary Condition Method (EBCM), e.g. [2,3], electric and magnetic fields are expanded in terms of a complete set of Vector Spherical Wave Functions (VSWFs). The coefficients of such expansions are proportional to what are called the beam shape coefficients (BSCs) $g^m_{n,TM}$ and $g^m_{n,TE}$ [4], TM for the Transverse Magnetic polarization and TE for the Transverse Electric polarization. They reduce to simpler expressions called special BSCs g_n in the case of an on-axis axisymmetric beam, and eventually to a simple constant phase term in the case of a plane wave [5]. One of the issues when dealing with any type of beam is the evaluation of these coefficients.

Since the pioneering papers of Durnin [6,7], there has been increasing interest in scattering by Bessel beams, and therefore in the computations of their BSCs. Cizmar et al. [8] developed a non-paraxial (vectorial) description of the Bessel beam electric and magnetic fields using an angular spectrum of plane waves approach, taking the form of a quadrature over an azimuthal angle β, with closed form expressions for the spectral components. In the case of a rotationally symmetric beam, i.e. when there is no dependence on β, the fields can be expressed in closed form in terms of cylindrical coordinates (z, ρ, φ). The BSCs can be obtained by double quadratures of the fields over angular spherical coordinates, which is the original method used to evaluate the BSCs in the GLMT in spherical coordinates, e.g. [9] and references therein, and [1, pp. 40–47] including a comparison between the formulations of Gouesbet et al. [10,11] and of Barton et al. [12,13]. These double quadratures reduce to single quadratures in the case of a zeroth-order Bessel beam [8], while numerical evaluations of the double quadratures are available from Preston et al. [14]. Milne et al. [15] studied the transverse

* Corresponding author.
E-mail address: gouesbet@coria.fr (G. Gouesbet).

http://dx.doi.org/10.1016/j.jqsrt.2016.06.039

particle dynamics in a zeroth-order Bessel beam, and similarly indicated the possible simplification of the BSCs from double quadratures to single quadratures. But, owing to the time-consuming character of this integration, they preferred to rely on a geometrical optics model.

These results were improved upon by Taylor and Love [16] who succeeded in deriving an analytic result for the BSCs that does not require the numerical evaluation of any integral. Chen et al. [17,18] afterward extended these results to the case of Bessel beams of arbitrary order and polarization, with an application to optical binding. Similar analytic expressions for BSCs were obtained by Ma and Li [19] who studied the scattering of an unpolarized Bessel beam by a sphere. Wang et al. [20] dealt with a dynamical and phase-diagram study on stable optical pulling force in zeroth- and higher-order Bessel beams, Lock [21] dealt with a discussion of the angular spectrum and localized model of Davis type beams, including a discussion of zeroth-order Bessel beams both on-axis and off-axis, and Song et al. [22] dealt with optical forces on a large sphere illuminated by a zeroth-order Bessel beam and with comparisons between the ray optics method and generalized Lorenz–Mie theory.

In the case of Gaussian beams, described by electric and magnetic field expressions which do not satisfy Maxwell's equations [23,24], the most efficient method for evaluating the BSCs has been the localized approximation, which may be presented under different formulations which describe fields that are closely similar to the original beam, and satisfy Maxwell's equations. A review of localized approximations is available from [25], which is complemented by [26,27]. The most general and complete rigorous justification of localized approximations for evaluating the beam fields in spherical coordinates using the N-beam approach is available from [28] which, in contrast with earlier justifications which were devoted to Gaussian beams, is valid for “arbitrary shaped beams”. This topic was also pursued using the angular spectrum of plane waves in [21]. These arbitrary shaped beams contain the phase factor $\exp(\pm ikz)$ for propagation along the z-axis. This does not imply that the speed of the laser beam is the speed of light c. It is actually smaller than c [29]. Unfortunately, Bessel beams do not fall under the aforementioned class of “arbitrary shaped beams” considered in [28]. This is because their propagation term is $\exp(\pm ikz\cos\alpha)$, with α being the half-cone or axicon angle. They have an angular spectrum of plane waves, all tilted by the same angle α with respect to the z-axis, and therefore possess a speed along this axis which is equal to $c\cos\alpha$.

Several papers examining scattering by a Bessel beam used a localized approximation (more specifically an integral localized approximation variant, see [27] for details regarding the terminology). Ambrosio and Hernandez described zeroth-order [30] and higher-order Bessel beams [31] by using this approximation, with applications to optical trapping. Qu et al. [32] dealt with the electromagnetic scattering of a zeroth-order Bessel beam by a uniaxial anisotropic sphere located in an off-axis Bessel beam, Li et al. [33] dealt with the analysis of radiation pressure force exerted on a biological cell induced by a high-order Bessel beam, Li et al. [34] dealt with the scattering of an axicon-generated Bessel beam by a sphere, and Chen et al. [35] dealt with the scattering of a zeroth-order Bessel beam by a concentrically coated sphere.

The use of localized approximations for Bessel beams is *in practice* valid when the axicon angle α is relatively small, as is well exemplified in Figs. 2a–b of Li et al. [34] who compared original expressions in terms of coordinates and reconstructed expressions using the localized BSCs for electric amplitudes, and reconstructed the spatial distributions in their Figs. 3a–i, with an axicon angle of 1°. However, it is important to be aware that significant errors may occur in the localized approximation of a Bessel beam when the axicon angle becomes sufficiently large. The origin of this fact relies on the presence of $\cos\alpha$ in the propagation term of Bessel beams, which does not occur in the case of the $\exp(\pm ikz)$-type beams considered in [28]. The deleterious consequence of the $\cos\alpha$-term is well illustrated in [36] in which it is shown that the N-beam procedure for deriving the localized approximation that was successfully used in [28] for $\exp(\pm ikz)$-type fails for Bessel beams.

This paper is organized as follows. Section 2 deals with rigorous closed form expression for the BSCs of a zeroth-order on-axis Bessel beam which is given by Eq. (9). Section 3 derives localized approximation to these BSCs, which is given by Eq. (26). Section 4 compares both expressions and numerically exhibits the fact that localized results increasingly depart from the rigorous expressions as the axicon angle increases. Section 5 presents a qualitative discussion of these results, and also serves as a conclusion.

2. Rigorous beam shape coefficients in closed form

Let us consider a Cartesian coordinate system $O_B uvw$ attached to the Bessel beam, with a time dependence of the form $\exp(-i\omega t)$ which is opposite to the convention usually used in GLMTs. The beam propagates from negative w to positive w. We then rely on Eqs. (49a)–(49f) of [21] describing, as an example, the Davis symmetrized fields of a zeroth-order Bessel beam, which are equivalent to Eqs. (14)–(19) from Mishra [37]. The electric field components E_i and magnetic field components H_i, $i = u, v, w$, are:

$$E_u = E_0\left\{\frac{1}{2}\left(1+\cos\alpha-\frac{\sin^2\alpha}{2}\right)J_0(k\rho\sin\alpha) + \frac{\sin^2\alpha}{4}\cos(2\varphi)J_2(k\rho\sin\alpha)\right\}\exp(ikw\cos\alpha) \tag{1}$$

$$E_v = E_0\frac{\sin^2\alpha}{4}\sin(2\varphi)J_2(k\rho\sin\alpha)\exp(ikw\cos\alpha) \tag{2}$$

$$E_w = -iE_0\frac{\sin\alpha}{2}(1+\cos\alpha)\cos\varphi J_1(k\rho\sin\alpha)\exp(ikw\cos\alpha) \tag{3}$$