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Semi-analytical solution to arbitrarily shaped beam scattering

Wenjie Wang, Huayong Zhang*, Yufa Sun

Key Lab of Intelligent Computing and Signal Processing, Ministry of Education, Anhui University, Hefei, Anhui 230039, PR China

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ABSTRACT

Based on the field expansions in terms of appropriate spherical vector wave functions and the method of moments scheme, an exact semi-analytical solution to the scattering of an arbitrarily shaped beam is given. For incidence of a Gaussian beam, zero-order Bessel beam and Hertzian electric dipole radiation, numerical results of the normalized differential scattering cross section are presented to a spheroid and a circular cylinder of finite length, and the scattering properties are analyzed concisely.

1. Introduction

One of the basic problems to investigate the interaction between electromagnetic (EM) waves and isotropic media is to describe the EM scattering by arbitrarily shaped isotropic objects. There has been a rather large literature on the EM scattering by isotropic particles, for incidence of a plane wave or a shaped beam. As the well-known analytical and semi-analytical solutions, the generalized Lorenz–Mie theory (GLMT) [1] and extended boundary condition method (EBCM) [2,3] are widely used, in which it is necessary to expand the incident EM fields in terms of the partial waves or plane waves. Using the surface integral equation (SIE) method, which expresses the scattered fields in terms of the equivalent EM currents by invoking Huygens's principle, Cui et al. analyzed the EM scattering form complex particles [4,5]. With the EBCM, a theoretical procedure is given in [6] to study the EM plane wave scattering by non-axisymmetric particles. This paper, based on [6], is devoted to the presentation of an alternative SIE procedure for the research on the scattering of an arbitrarily shaped beam by a dielectric particle.

The body of this paper is organized as follows. In Section 2, the theoretical procedure for the determination of the scattered fields of an arbitrarily shaped beam by a dielectric particle is provided. Section 3 presents the numerical results and discussions of the scattering characteristics of a Gaussian beam, zero-order Bessel beam (ZOBB) and Hertzian electric dipole (HED) radiation. The present work is concluded in Section 4.

2. Formulation

As illustrated in Fig. 1, a shaped beam propagates in free space and from the negative z' to the positive z' axis in its own Cartesian coordinate system $O'x'y'z'$ (shaped beam coordinate system). An accessory system $Ox''y''z''$ that is parallel to $O'x'y'z'$ is introduced, with origin O having the Cartesian coordinates (x_0, y_0, z_0) in $O'x'y'z'$. A dielectric particle is natural to the system $Oxyz$ which is obtained by rotating $Ox''y''z''$ through a single Euler angle β [7]. In this paper, an $\exp(-i\omega t)$ time variation is assumed.

* Corresponding author.
E-mail address: hyzhang0905@163.com (H. Zhang).

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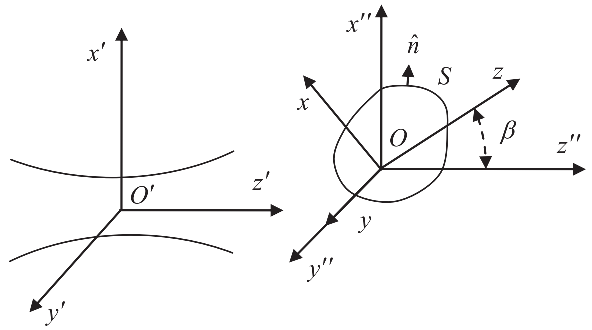

Fig. 1. A dielectric particle illuminated by a shaped beam.

According to the radiation condition of an outgoing wave, an appropriate expansion of the scattered fields in terms of the spherical vector wave functions (SVWFs) can be given by

$$\mathbf{E}^s = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\alpha_{mn}\mathbf{M}_{mn}^{r(3)}(k)+\beta_{mn}\mathbf{N}_{mn}^{r(3)}(k)\right] \tag{1}$$

$$\mathbf{H}^s = -i\frac{k}{\omega\mu}E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\alpha_{mn}\mathbf{N}_{mn}^{r(3)}(k)+\beta_{mn}\mathbf{M}_{mn}^{r(3)}(k)\right] \tag{2}$$

where α_{mn}, β_{mn} are the unknown expansion coefficients to be determined, and k is the free space wave number.

The EM fields within a dielectric particle (internal fields) can correspondingly be expanded in terms of the SVWFs, as follows:

$$\mathbf{E}^{\text{int}} = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\delta_{mn}\mathbf{M}_{mn}^{r(1)}(k')+\gamma_{mn}\mathbf{N}_{mn}^{r(1)}(k')\right] \tag{3}$$

$$\mathbf{H}^{\text{int}} = -i\frac{k'}{\omega\mu}E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\delta_{mn}\mathbf{N}_{mn}^{r(1)}(k')+\gamma_{mn}\mathbf{M}_{mn}^{r(1)}(k')\right] \tag{4}$$

where $k' = k\tilde{n}$, and $\tilde{n}$ is the refractive index of the material of the dielectric particle relative to that of free space.

In the method of moments (MoM) scheme, Eqs. (1)–(4) can be interpreted that the expansions of the scattered and internal fields are obtained using appropriate SVWFs as basis functions.

The boundary conditions at the dielectric particle's surface S require that the tangential components of the EM fields be continuous

$$\hat{n} \times (\mathbf{E}^s + \mathbf{E}^{inc}) = \hat{n} \times \mathbf{E}^{\text{int}} \tag{5}$$

$$\hat{n} \times (\mathbf{H}^s + \mathbf{H}^{inc}) = \hat{n} \times \mathbf{H}^{\text{int}} \tag{6}$$

where $\mathbf{E}^{inc}$ and $\mathbf{H}^{inc}$ respectively represent the electric and magnetic fields of the incident shaped beam, and $\hat{n}$ denotes the outward normal to the particle surface S.

A substitution of Eqs. (1)–(4) into Eqs. (5) and (6) leads to

$$\hat{n} \times E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\alpha_{mn}\mathbf{M}_{mn}^{r(3)}(k)+\beta_{mn}\mathbf{N}_{mn}^{r(3)}(k)\right] + \hat{n} \times \mathbf{E}^{inc}$$
$$= \hat{n} \times E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\delta_{mn}\mathbf{M}_{mn}^{r(1)}(k')+\gamma_{mn}\mathbf{N}_{mn}^{r(1)}(k')\right] \tag{7}$$

$$\hat{n} \times kE_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\alpha_{mn}\mathbf{N}_{mn}^{r(3)}(k)+\beta_{mn}\mathbf{M}_{mn}^{r(3)}(k)\right] + i\omega\mu\hat{n}$$
$$\times \mathbf{H}^{inc} = \hat{n} \times k'E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\delta_{mn}\mathbf{N}_{mn}^{r(1)}(k')+\gamma_{mn}\mathbf{M}_{mn}^{r(1)}(k')\right] \tag{8}$$

Eqs. (7) and (8) are multiplied by $\mathbf{M}_{m'n'}^{r(1)}(k')$, $\mathbf{N}_{m'n'}^{r(1)}(k')$ (weighting functions) respectively and integrated over the surface S following the MoM procedure. By assuming the incident fields $\mathbf{E}^{inc}$ and $\mathbf{H}^{inc}$ to be known, the following equations can be obtained.

$$-\oint_S \mathbf{E}^{inc} \times \mathbf{M}_{m'n'}^{r(1)}(k') \cdot \hat{n}dS = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\alpha_{mn}\oint_S \mathbf{M}_{mn}^{r(3)}(k)\right.$$
$$\left. + \beta_{mn}\oint_S \mathbf{N}_{mn}^{r(3)}(k)\right] \times \mathbf{M}_{m'n'}^{r(1)}(k') \cdot \hat{n}dS$$
$$-E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\delta_{mn}\oint_S \mathbf{M}_{mn}^{r(1)}(k') + \gamma_{mn}\oint_S \mathbf{N}_{mn}^{r(1)}(k')\right]$$
$$\times \mathbf{M}_{m'n'}^{r(1)}(k') \cdot \hat{n}dS \tag{9}$$

$$-\oint_S \mathbf{E}^{inc} \times \mathbf{N}_{m'n'}^{r(1)}(k') \cdot \hat{n}dS = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\alpha_{mn}\oint_S \mathbf{M}_{mn}^{r(3)}(k)\right.$$
$$\left. + \beta_{mn}\oint_S \mathbf{N}_{mn}^{r(3)}(k)\right] \times \mathbf{N}_{m'n'}^{r(1)}(k') \cdot \hat{n}dS$$
$$-E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\delta_{mn}\oint_S \mathbf{M}_{mn}^{r(1)}(k') + \gamma_{mn}\oint_S \mathbf{N}_{mn}^{r(1)}(k')\right]$$
$$\times \mathbf{N}_{m'n'}^{r(1)}(k') \cdot \hat{n}dS \tag{10}$$

$$-i\omega\mu\oint_S \mathbf{H}^{inc} \times \mathbf{M}_{m'n'}^{r(1)}(k') \cdot \hat{n}dS = kE_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\alpha_{mn}\oint_S \mathbf{N}_{mn}^{r(3)}(k)\right.$$
$$\left. + \beta_{mn}\oint_S \mathbf{M}_{mn}^{r(3)}(k)\right] \times \mathbf{M}_{m'n'}^{r(1)}(k') \cdot \hat{n}dS$$
$$-k'E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\delta_{mn}\oint_S \mathbf{N}_{mn}^{r(1)}(k') + \gamma_{mn}\oint_S \mathbf{M}_{mn}^{r(1)}(k')\right]$$
$$\times \mathbf{M}_{m'n'}^{r(1)}(k') \cdot \hat{n}dS \tag{11}$$

$$-i\omega\mu\oint_S \mathbf{H}^{inc} \times \mathbf{N}_{m'n'}^{r(1)}(k') \cdot \hat{n}dS = kE_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\alpha_{mn}\oint_S \mathbf{N}_{mn}^{r(3)}(k)\right.$$
$$\left. + \beta_{mn}\oint_S \mathbf{M}_{mn}^{r(3)}(k)\right] \times \mathbf{N}_{m'n'}^{r(1)}(k') \cdot \hat{n}dS$$
$$-k'E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[\delta_{mn}\oint_S \mathbf{N}_{mn}^{r(1)}(k') + \gamma_{mn}\oint_S \mathbf{M}_{mn}^{r(1)}(k')\right]$$
$$\times \mathbf{N}_{m'n'}^{r(1)}(k') \cdot \hat{n}dS \tag{12}$$

If two vector fields $\mathbf{P}$ and $\mathbf{Q}$ satisfy the vector wave equation $\nabla \times \nabla \times \mathbf{A} - k'^2\mathbf{A} = 0$ ($\mathbf{A}$ being a vector function), as described in [6] the following formulation can be obtained from the vector Green's theorem

$$\oint_S (\mathbf{Q} \times \nabla \times \mathbf{P}) \cdot \hat{n}dS = \oint_S (\mathbf{P} \times \nabla \times \mathbf{Q}) \cdot \hat{n}dS \tag{13}$$

Since the SVWFs $[\mathbf{M},\ \mathbf{N}]_{mn}^{r(1)}(k')$ and $[\mathbf{M},\ \mathbf{N}]_{m'n'}^{r(1)}(k')$ satisfy the vector wave equation, from Eq. (13) and the