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Numerical calculation of light scattering from metal and dielectric randomly rough Gaussian surfaces using microfacet slope probability density function based method

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ABSTRACT

Light scattering from randomly rough surfaces is of great significance in various fields such as remote sensing and target identification. As numerical methods can obtain scattering distributions without complex setups and complicated operations, they become important tools in light scattering study. However, most of them suffer from huge computing load and low operating efficiency, limiting their applications in dynamic measurements and high-speed detections. Here, to overcome these disadvantages, microfacet slope probability density function based method is presented, providing scattering information without computing ensemble average from numerous scattered fields, thus it can obtain light scattering distributions with extremely fast speed. Additionally, it can reach high-computing accuracy quantitatively certificated by mature light scattering computing algorithms. It is believed the provided approach is useful in light scattering study and offers potentiality for real-time detections.

1. Introductions

Surfaces of most natural and artificial materials can be treated as randomly rough ones in visible light scale, thus light scattering from them is basis of various theoretical studies and practical applications as imaging, surface detecting and remote sensing, *etc.* [1–3]. In order to deeply understand light scattering from such surfaces, both experimental and numerical studies have been proposed [4–10]. Compared to experimental research, numerical computation is capable of revealing scattering fields with high accuracy and is less limited by experimental conditions, therefore, it remains as an important tool in light scattering analysis.

To quantitatively study light scattering from surfaces, various numerical methods are provided. Method of moments (MoM) which can obtain light scattering fields without any approximations is one of the classical techniques [11]. It transforms electromagnetic equations into matrix systems of linear equations which can be solved using computers. Actually, it uses orthogonal expansions to reduce the integral equation problem to linear equations. Simonsen et al. used MoM to study both intensity and polarization information of light scattering from randomly rough surfaces [12–15]. Tsang et al. applied sparse-matrix canonical grid method to study scattering characteristics from scatterers and rough surfaces [16–18]. However, MoM is often computation-intensive and time-consuming. To improve computing efficiency, approximations were introduced. Maradudin et al. adopted small perturbation method to study light scattering from randomly rough surfaces and gratings [19,20]. Buckius et al. proposed methods such as geometric optics approximation for reflection calculation from randomly rough surfaces [21–23]. Among them, Kirchhoff approximation (KA) method is a widely used approach [24–36]. In KA, the rough surface is approximated to collections of tangent planes; then, local field of light can be computed according to the law of reflection; finally, classical Stratton-Chu equation is adopted to solve the scattered fields. Light scattering distributions from different kinds of surfaces, as randomly rough surfaces [25–31], surfaces with infinite slopes [32,33], and grooves [34], were calculated by different research groups, especially by Bruce et al. Moreover, improved KA method was also proposed to expand its application scope [35,36]. Additionally, small-slope approximation was proposed to unify both small-perturbation model and KA [37–41]. Guo et al. used this method mainly in electromagnetic scattering testing [42,43], Wang and Broschat applied it in scattering from randomly rough surfaces with low grazing angles [44–46].

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Compared to MoM, these approximation based methods not only keep high-calculation accuracy in light scattering, but also have faster computing speed. However, computing efficiency improvement is still not evident, it is because most of them still need ensemble average of light scattering calculation from multiple surfaces. Only with this process, stable light scattering fields can be obtained. Unfortunately, the repeated process leads to heavy computing load and low operating efficiency, limiting potential applications of these methods in dynamic measurements and high-speed detections.

In order to realize rapid and accurate light scattering calculation, here, microfacet slope probability density function based method is presented. It calculates light scattering directly based on microfacet slope probability density function [47,48], which avoids ensemble average computation, thus, it can provide light scattering information with extremely fast speed. Additionally, it can reach high precision in scattering computation certificated by mature techniques as MoM and KA. Moreover, as a quasi-quantitative numerical calculation approach, it can compute light scattering distributions from rough surfaces in various conditions as different polarizations, surface materials, incidence angles, surfaces fluctuations. Considering these advantages, we believe the designed method offers potentiality for real-time detections in both theoretical study and practical measurements.

2. Theory

Fig. 1 shows the scheme of light scattering from one dimensional randomly rough surface. Scattering light is integration of waves from multiple microfacets located in the randomly rough surface. Since sizes of microfacets are rather small, they can be approximately treated as small slopes as shown in the embedded figure. **X** and **Z** are perpendicular axes: randomly rough surface is mainly located on the **X** axis, combining with **Z** axis, both incidence and scattering light can be defined: $\mathbf{k_i}$ and $\mathbf{k_s}$ are wave vectors of both incidence and scattering waves, respectively; similarly, θ_i and θ_s are incidence and scattering angles. Far field of light scattering is composed of both incidence and local fields of all microfacets. As incidence angle is determined, contributors to scattering field are those microfacets which maintain incidence angle θ_i equals to scattering angle θ_s. Moreover, in Fig. 1, it is obvious that $\theta_{im}=(\theta_i+\theta_s)/2$ and $\alpha=(\theta_i-\theta_s)/2$. Slope of microfacet s is shown in Eq. (1), in which z represents the surface profile.

$$s=\frac{dz}{dx}=\tan\alpha=\tan\left(\frac{\theta_i-\theta_s}{2}\right) \tag{1}$$

Here, in this paper, only randomly rough Gaussian surfaces are considered, whose height-distributions and correlation functions have the Gaussian form, their slope probability density function is illustrated in Eq. (2), in which average slope is μ_s and root-mean-square slope is δ_s.

$$p(s)=\frac{1}{\sqrt{2\pi}\delta_s}\exp\left(-\frac{(s-\mu_s)^2}{2\delta_s{}^2}\right) \tag{2}$$

Fig. 2 shows slope probability density functions with different roughness according to Eq. (2): incidence angles are 0°, 10° and 20°, respectively, correlation lengths T of randomly rough surfaces are set as 4λ and root-mean-square heights δ range from 0.3λ to 1.1λ, λ is wavelength of the incidence light. The integral of slope probability density function along complete scattering angle range is 1.

As δ is low, indicating the surface is smooth, slope probability density functions are mostly fastened on specular locations; however, as δ increases, slope probability density functions broaden obviously and their peak values decrease. Moreover, locations of these slope probability density functions shift with different incidence angles. Generally, the center of slope probability density function is at its specular direction according to the incidence light. It is worth noting that light scattering depends on both material and roughness of randomly rough surfaces. Slope probability density function describes roughness of surfaces, and reflectance is determined by refractive index of the material through Fresnel equations: both amplitude reflection coefficients for s- and p-polarization light are described by Eq. (3).

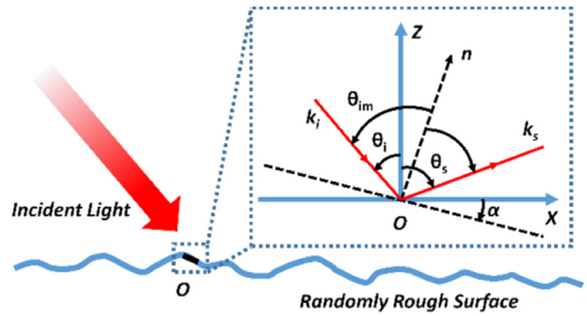

Fig. 1. Scheme of light scattering from a randomly rough surface. When the incidence light is at the left of the z axis (counter-clockwise from **z** to $\mathbf{k_i}$), the sign of θ_i is positive; while when the scattering light is at the right of the z axis (clockwise from **z** to $\mathbf{k_s}$), the sign of θ_s is positive.

$$r_s=\frac{\cos\theta_{im}-n_0\cos\theta_{tm}}{\cos\theta_{im}+n_0\cos\theta_{tm}}$$
$$r_p=\frac{n_0\cos\theta_{im}-\cos\theta_{tm}}{n_0\cos\theta_{im}+\cos\theta_{tm}} \tag{3}$$

In Eq. (3), θ_{im} and θ_{tm} are incidence and refraction angles, respectively, n_0 is refractive index ratio between incidence and refraction media. Based on Snell's law as Eq. (4), amplitude reflection coefficients for s- and p-polarization can be derived in Eq. (5).

$$\sin\theta_{tm}=\frac{1}{n_0}\sin\theta_{im}$$
$$\cos\theta_{tm}=\sqrt{1-\frac{1}{n_0^2}\sin^2\theta_{im}} \tag{4}$$

$$r_s=\frac{\cos\theta_{im}-\sqrt{n_0{}^2-\sin^2\theta_{im}}}{\cos\theta_{im}+\sqrt{n_0{}^2-\sin^2\theta_{im}}}$$
$$r_p=\frac{n_0^2\cos\theta_{im}-\sqrt{n_0^2-\sin^2\theta_{im}}}{n_0^2\cos\theta_{im}+\sqrt{n_0^2-\sin^2\theta_{im}}} \tag{5}$$

As mentioned above, Eq. (6) can be derived by substituting $\theta_{im}=(\theta_i+\theta_s)/2$ into Eq. (5).

$$r_p=\frac{n_0\cos((\theta_i+\theta_s)/2)-\sqrt{n_0^2-\sin^2((\theta_i+\theta_s)/2)}}{n_0\cos((\theta_i+\theta_s)/2)+\sqrt{n_0^2-\sin^2((\theta_i+\theta_s)/2)}}$$
$$r_s=\frac{\cos((\theta_i+\theta_s)/2)-\sqrt{n_0^2-\sin^2((\theta_i+\theta_s)/2)}}{\cos((\theta_i+\theta_s)/2)+\sqrt{n_0^2-\sin^2((\theta_i+\theta_s)/2)}} \tag{6}$$

Eq. (6) is the amplitude reflection coefficient, and the reflectance can be written as $R=|r|^2$ shown in Eq. (7).