Journal of Quantitative Spectroscopy & Radiative Transfer 194 (2017) 31–46

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Assessment of different radiative transfer equation solvers for combined natural convection and radiation heat transfer problems

Yujia Sun [a,b], Xiaobing Zhang [a,*], John R. Howell [b]

[a] Nanjing University of Science and Technology, Nanjing, Jiangsu 210094, China
[b] The University of Texas at Austin, Austin, TX 78712, USA

ARTICLE INFO

Article history:
Received 29 November 2016
Received in revised form
9 March 2017
Accepted 9 March 2017
Available online 11 March 2017

Keywords:
Natural convection
Radiation
Combined heat transfer

ABSTRACT

This work investigates the performance of the DOM, FVM, P_1, SP_3 and P_3 methods for 2D combined natural convection and radiation heat transfer for an absorbing, emitting medium. The Monte Carlo method is used to solve the RTE coupled with the energy equation, and its results are used as benchmark solutions. Effects of the Rayleigh number, Planck number and optical thickness are considered, all covering several orders of magnitude. Temperature distributions, heat transfer rate and computational performance in terms of accuracy and computing time are presented and analyzed.

1. Introduction

Combined heat transfer problems with radiation are found in many industrial devices, especially those with high temperature. Overall thermal analysis must take conduction, convection and radiation into consideration. For coupled problems, many solvers for the radiative transfer equation (RTE) have been used by different investigators. Also, some of these methods are incorporated into commercial or open source codes, but how to choose among these methods for different problems is not clear, so they leave the choice of methods up to the user. Numerical simulations of combined problems often start from zero or uniform internal temperature with different wall temperature (discontinuous or continuous). Thus, different RTE solvers coupled with the energy equation iteration may lead to quite different convergence behavior; that is, combined problem convergence may rely largely on the performance of RTE solvers. The purpose to this work is to provide insight into the factors that are necessary to choose an appropriate RTE solver for a combined natural convection and radiation problem in a square cavity.

There exist many papers on combined natural convection and radiation in a square cavity. Yücel [1] studied natural convection-radiation interactions in externally and internally heated enclosures, while only one set of Rayleigh number and Planck number was considered for the externally heated condition. Yücel concluded that the P_1 method was slower than S_4 DOM for optically thin media. Tan and Howell [2] considered this problem with an absorbing, emitting and isotopically scattering medium for three Rayleigh numbers and a wide range of radiation-conduction parameter. Results showed that the presence of radiation increases the bulk temperature of the fluid and has a significant influence on the fluid flow and temperature distributions, while scattering albedo has small effect on the heat transfer. Mondal and Mishra [3] used the Lattice Boltzmann method (LBM) for the heat and fluid solver and FVM for the RTE. They concluded that the flow field was significantly affected by radiation at a high Ra, and the extinction coefficient (optical thickness) has a pronounced effect on the temperature distribution. Lari [4] et al. considered a broad range of Rayleigh numbers (10^2–10^6) and optical thicknesses (0–100), and found that overall Nusselt number on the walls decreases with increasing optical thickness for all Rayleigh numbers. Moufekkir et al. [5] adopted a hybrid thermal lattice Boltzmann method for the combined problem in a tilted square cavity. They reached a similar conclusion that increasing the optical thickness causes a decrease in the heat transfer and radiation induces an increase of the temperature. They also found that inclination angle has a strong effect on the structure of isotherms and streamlines. Ibrahim et al. [6] considered non-gray effects and turbulent flow in this combined problem. For their configuration, gas radiation has little influence on the flow structure, but it tends to stabilize the flow and homogenize the temperature field.

The above researchers generally used a specific RTE solver for the combined problem, because their focus is on a specific numerical method or problem. Apparently there was no major concern about the criteria for the choice of the RTE solver. Tencer [7] analyzed the advantages and disadvantages of different RTE solvers for conjugate heat transfer problems. Mishra [8] compared

* Corresponding author.

http://dx.doi.org/10.1016/j.jqsrt.2017.03.022
0022-4073/

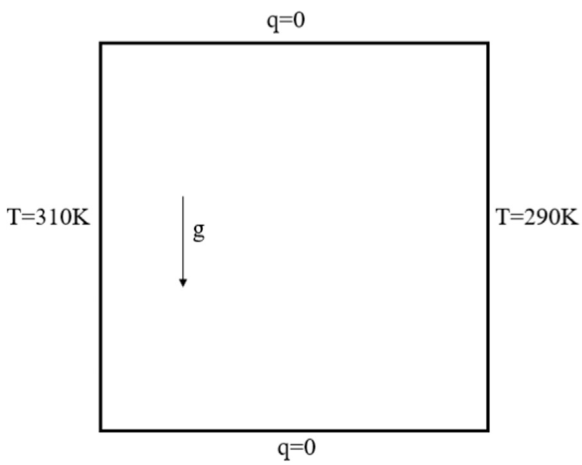

Fig. 1. Problem geometry.

DTM, DOM and FVM methods for transient combined conduction and radiation heat transfer and found that DTM is the most time consuming while DOM is most efficient. Sun [9] performed a detailed analysis of different RTE solvers for combined conduction and radiation problems. Results show that the FVM is efficient for cases of optical thickness smaller than 5.0, while P_1 and SP_3 are very slow. There are also papers dealing with combinations of different energy solvers and RTE solvers for combined problems, such as the spectral collocation method [10,11], natural element method [12,13], meshless method [14,15], lattice Boltzmann method (LBM) coupled with DOM and FVM [16–19], and LBM for both the energy equation and RTE [20]. These works mainly focus on developing new solvers for combined problems while how to choose an appropriate method for particular combined convection and radiation heat transfer problems remains to be investigated.

To eliminate this gap, this paper aims to investigate the performance of DOM, FVM, P_1, SP_3 and P_3 for a 2D combined natural convection and radiation heat transfer problem. Additionally, Monte Carlo methods (MC) are also coupled with the heat and fluid flow to generate benchmark solutions. Heat flux results are presented along with detailed results for accuracy and computational time.

2. Mathematical formulation

In this paper, we consider a two-dimensional, steady, laminar flow with the Boussinesq approximation for the buoyancy force.

2.1. Governing equations for natural convection

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$
$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$
$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\beta(T-T_0)$$
$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-\frac{1}{\rho c_p}\nabla\cdot\mathbf{q}_r \tag{1}$$

where u is the horizontal velocity, v is the vertical velocity, T is the temperature, ν is the dynamic viscosity, β is the thermal expansion coefficient, α is the thermal diffusivity, T_0 is the average of the hot wall and cold wall temperatures. The radiative heat source can be obtained as:

$$\nabla\cdot\mathbf{q}_r=\kappa(4\pi I_b-G) \tag{2}$$

where I_b is the blackbody emission intensity of the medium and G is the incident radiation, which needs to be determined from solving the radiative transfer equation.

2.2. Radiative transfer equation

The conservation equation of radiation in an absorbing, emitting and scattering medium can be written as:

$$\Omega\cdot\nabla I=-\beta_r I+\kappa I_b+\frac{\sigma_s}{4\pi}\int_{4\pi}I(\Omega')\,\Phi(\Omega,\Omega')\mathrm{d}\Omega' \tag{3}$$

where $\beta_r=\kappa+\sigma_s$ is the extinction coefficient, κ is the absorption coefficient, σ_s is the scattering coefficient. Wavenumber subscript is not used here as the medium is assumed gray in this work. Those solvers used in this work will be briefly described in next section. Scattering is not considered in this work, so the RTE is simplified to:

$$\Omega\cdot\nabla I=-\beta_r I+\kappa I_b \tag{4}$$

now $\beta_r=\kappa$.

3. Numerical methods

In this section, numerical methods used for solving the combined natural convection and radiation heat transfer problem will be briefly described.

3.1. Problem description

We consider a square enclosure here with top and bottom walls insulated, and two vertical walls at fixed temperatures as shown in Fig. 1. The left wall is at high temperature, and the right wall is at low temperature. Gravity is in the negative Y direction. Wall emissivities are set to 1 for all four walls.

3.2. Important dimensionless numbers

The results are presented in terms of dimensionless numbers which govern the flow and heat transfer characteristics of this problem.

Rayleigh Number:

$$Ra=\frac{g\beta\Delta TL^3}{\nu\alpha} \tag{5}$$

Planck Number:

$$Pl=\frac{k}{\sigma T^3 L} \tag{6}$$

where k is thermal conductivity, which should not be mistaken as absorption coefficient κ.

Optical thickness:

$$\tau=\kappa L \tag{7}$$

Convective Nusselt Number:

$$Nu_c=\frac{q_c L}{k\Delta T} \tag{8}$$

Radiative Nusselt Number: