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An analysis of the symmetry issue in the ℓ-distribution method of gas radiation in non-uniform gaseous media

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ABSTRACT

The recently proposed ℓ-distribution/ICE (Iterative Copula Evaluation) method of gas radiation suffers from symmetry issues when applied in highly non-isothermal and non-homogeneous gaseous media. This problem is studied in a detailed theoretical way. The objective of the present paper is: 1/to provide a mathematical analysis of this problem of symmetry and, 2/to suggest a decisive factor, defined in terms of the ratio between the narrow band Planck and Rosseland mean absorption coefficients, to handle this issue. Comparisons of model predictions with reference LBL calculations show that the proposed criterion improves the accuracy of the intuitive ICE method for applications in highly non-uniform gases at high temperatures.

1. Introduction

The recently proposed ℓ-distribution method was introduced in Ref. [1] for the approximate modeling of the radiative properties of non-uniform gaseous media. It is built on the following ideas: 1/ the uniform ℓ-distribution model is founded on a statistical method coupled with results taken from the *k*-moments method [1]. In practice, it mostly consists of a tabulation of a function that maps the LBL emissivities or absorptivities of uniform (homogeneous isothermal) gaseous paths averaged over spectral bands with an approximate model. The ℓ-distribution method was shown in Ref. [1] to achieve LBL accuracy in uniform situations at very small computational expenses, 2/ an iterative scheme that propagates the radiation along a non-uniform layer in a step-by-step manner. This process is called the ICE (Iterative Copula Evaluation) scheme in the ℓ-distribution method (it will be from now on referred to as ICE so as to abridge the notations). The principle of the technique to treat path non-uniformities is based on a scaling approximation similar to the one described by Godson in 1953 [2], almost at the same time as the widely used Curtis-Godson approximation. Godson's technique was "rediscovered" in the early 70s by Weinreb and Neuendorffer [3] and slightly reformulated under the name EGA in Ref. [4]. The EGA technique has applications in radiative heat transfer in the atmosphere, and is used for instance in the codes JURASSIC [5] and BANDPAK [6]. In the recent book by Young [7], this method to propagate the radiation along a non-uniform path is called the "Godson-Weinreb-Neuendorffer's method" (GWN). The ICE scheme is similar to the GWN method in the sense that both techniques are based on the definition of effective scaling coefficients. However, this approximation is only one possibility to treat path non-uniformities in the ℓ-distribution approach: a more general technique was described in Ref. [8].

The main problem with the ICE method is that it suffers from symmetry issue: this means that if one considers a non-uniform path represented as the juxtaposition of two uniform sub-paths, this technique does not ensure the same results to be obtained if the calculation of band averaged transmissivities is performed from right to left or from left to right. A similar problem of symmetry was noticed by Godson [2] in the frame of the Elssasser's model. One can observe that symmetry issues are not a particular feature of the ICE or GWN methods as comparable problems were reported with the Lindquist-Simmons approximation in Refs. [7,9].

The aim of the present paper is to propose a simple and accurate method to handle the symmetry issue encountered in the ℓ-distribution/ICE method. The theoretical developments are extensively founded on the formulation used in the ℓ-distribution approach: accordingly, and although some results provided here are general, understanding Ref. [1] is highly recommended to follow the ideas and mathematical derivations developed in this paper. The solution proposed to choose a direction of application of the non-uniform approximation inside a non-isothermal and non-homogeneous layer is shown to agree with the intuitive approaches described by Godson [2], Weinreb and Neuendorffer [3]

Abbreviations: COG, Curve-Of-Growth; EGA, Emissivity Growth Approximation; GWN, Godson-Weinreb-Neuendorffer's method; ICE, Iterative Copula Evaluation; LBL, Line-By-Line; LS-ICE, Iterative Copula Evaluation with Levy Subordinators; SNB, Statistical Narrow Band model

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Nomenclature

C copula
Gr rank transmutation map
ℓ inverse of the transmission function (cm) as defined in Section 2.
L gas path length (cm)

Greek symbols

α absorptivity defined as 1-τ
β see Eq. (14).
δL small path length increment (cm)
κ_η spectral absorption coefficient (cm^{-1})
Λ inverse of the second order k-moment absorptivity model, see Eq. (16) (cm)
η wavenumber (cm^{-1})
τ transmission function; transmissivity

Subscripts

P Planck mean
R Rosseland mean
$\langle n \rangle$ related to the k-moment model at order n
n related to the n-th uniform sub-path along a non-uniform path
$1.n$ related to the $L_1 + L_2 + .. + L_n$ non-uniform path

Superscripts

e effective
$1,2$ state 1 or 2 of the gas
$\Delta\eta$ width of the spectral interval for the averaging of spectral properties

Other notation

$f \circ g$ represents the functional composition of functions f and g i.e. $f \circ g(x) = f[g(x)]$

and also discussed by Young [7] when restricted to Statistical Narrow Band formulations. The technique is assessed against Line-By-Line (LBL) calculations in situations representative of combustion applications (some of them are taken from Ref. [10]). This improved ICE scheme, called Levy Subordinated in order to employ the terminology used in Copula's theory that inspired the ℓ-distribution method, is shown: 1/ to provide results as accurate as the k-distribution model, 2/ to improve the recently proposed ICE scheme for highly non-uniform applications.

2. Principle of the ICE scheme

The main idea behind the ICE scheme is the following one. Let us consider a gas in a given thermophysical state ϕ_1. Small modifications of temperature, total pressure and species concentrations can be represented by a second state, ϕ_2. If the changes between states ϕ_1 and ϕ_2 are small, the spectral absorption coefficients in the two states are similar but not rigorously the same (line intensities and profiles depend on the state of the gas). However, it is possible, as a first approximation, to estimate the band averaged transmissivities of a path of length L in the gas in the state ϕ_2 by the transmissivity of the gas in the state ϕ_1. This requires defining an effective scaling coefficient between the spectra in the two states. More details are provided in Appendix A. This process then allows approximating transmissivities over non-uniform paths by transmissivities over equivalent isothermal and homogeneous layers. Fundamentally, these methods share many similarities with scaling approximations. Furthermore, one can notice that the ICE scheme was shown in Ref. [1] to coincide with the Curtis-Godson approximation at the optically thin and thick limits when applied to Statistical Narrow Band (SNB) models. A similar derivation was proposed in Ref. [11] for the GWN method.

More formally, let us consider a non-uniform layer discretized in two homogeneous isothermal sub-paths: the first one has a length L_1 and the gas is in the thermophysical state ϕ_1, the second path has a length L_2 and the state of the gas is ϕ_2. This is depicted in Fig. 1. The spectral absorption coefficients in the two layers are κ_η^1 and κ_η^2 respectively. They are assumed strictly positive over a

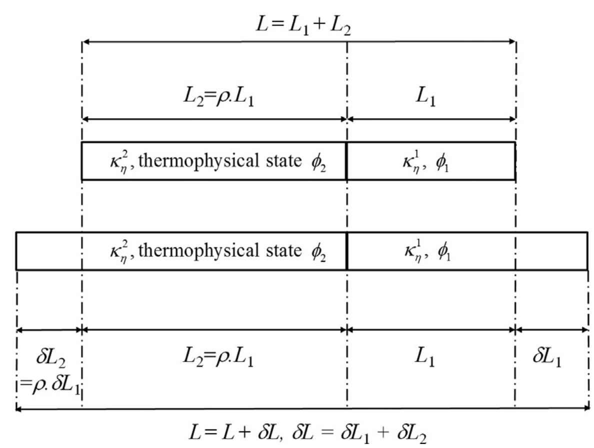

Fig. 1. Schematic representation of a non-uniform gaseous path.

narrow band $\Delta\eta$ over which the Planck function is constant (the problem is thus formulated here within the frame of narrow band approaches). Our objective is to evaluate the transmissivity of the non-uniform path $L = L_1 + L_2$ averaged over the band $\Delta\eta$. This transmissivity will be written $\tau_{12}^{\Delta\eta}(L_1, L_2)$. It is defined as:

$$\tau_{12}^{\Delta\eta}(L_1, L_2) = \frac{1}{\Delta\eta}\int_{\Delta\eta} \exp\left(-\kappa_\eta^1 L_1 - \kappa_\eta^2 L_2\right) d\eta \tag{1}$$

As shown in Appendix A, the ICE scheme introduced in Ref. [1] provides an approximation of $\tau_{12}(L_1, L_2)$ as:

$$\tau_{12}^{\Delta\eta}(L_1, L_2) \approx \tilde{\tau}_{12}^{\Delta\eta}(L_1, L_2) = \tilde{C}_{12}\left[\tau_1^{\Delta\eta}(L_1), \tau_2^{\Delta\eta}(L_2)\right] \tag{2}$$

where:

$$\tilde{C}_{12}(X_1, X_2) = \frac{1}{\Delta\eta}\int_{\Delta\eta} \exp\left(-\kappa_\eta^2\left[\ell_2(X_1) + \ell_2(X_2)\right]\right) d\eta \tag{3}$$

Function $\ell_2(X)$, $X \in [0, 1]$ in Eq. (3) is defined as the inverse of the transmissivity $\tau_2^{\Delta\eta}(L_2)$ over the uniform path L_2, viz. $\ell_2\left[\tau_2^{\Delta\eta}(L_2)\right] = L_2$.