Journal of Quantitative Spectroscopy & Radiative Transfer 189 (2017) 189–197

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Generalized source Finite Volume Method for radiative transfer equation in participating media

Biao Zhang, Chuan-Long Xu*, Shi-Min Wang

Key Laboratory of Energy Thermal Conversion and Control of Ministry of Education, School of Energy and Environment, Southeast University, Nanjing 210096, China

ARTICLE INFO

Article history:
Received 18 May 2016
Received in revised form
21 October 2016
Accepted 25 November 2016
Available online 29 November 2016

Keywords:
Radiative transfer equation
Finite Volume Method
Monte-Carlo method
Radial basis function

ABSTRACT

Temperature monitoring is very important in a combustion system. In recent years, non-intrusive temperature reconstruction has been explored intensively on the basis of calculating arbitrary directional radiative intensities. In this paper, a new method named Generalized Source Finite Volume Method (GSFVM) was proposed. It was based on radiative transfer equation and Finite Volume Method (FVM). This method can be used to calculate arbitrary directional radiative intensities and is proven to be accurate and efficient. To verify the performance of this method, six test cases of 1D, 2D, and 3D radiative transfer problems were investigated. The numerical results show that the efficiency of this method is close to the radial basis function interpolation method, but the accuracy and stability is higher than that of the interpolation method. The accuracy of the GSFVM is similar to that of the Backward Monte Carlo (BMC) algorithm, while the time required by the GSFVM is much shorter than that of the BMC algorithm. Therefore, the GSFVM can be used in temperature reconstruction and improvement on the accuracy of the FVM.

1. Introduction

At present, more than 70% of energy is generated from the combustion of fossil fuels [1]. Temperature is one of the most important parameters characterizing a combustion system. Improvements in combustion efficiency and pollution emission control require a comprehensive understanding of temperature distributions in furnaces or boilers [2]. In recent years, non-intrusive reconstruction technology has been explored intensively to obtain temperature readings through inverse radiation analysis using flame emission [3–6]. The visualization of 2D/3D temperature distributions from radiative energy images consists of two equally important tasks: the calculation of the radiative energy and the inverse of the temperature distribution [7].

For the first task, the radiative energy can be accurately determined by solving the radiative transfer equation (RTE). There are many methods that can be used to solve the RTE, such as Zone Method (ZM), Monte Carlo Method (MCM), Finite Element Method (FEM), Discrete Transfer Method (DTM), Discrete Ordinates Method (DOM), Ray Tracing Method (RTM), Finite Volume Method (FVM), Chebyshev Spectral Method (CSM), etc. [8–15]. The MCM is a special one. It is popular due to its simple coding and high parallel efficiency, it is easy to handle complex problems. It owns high accuracy with large enough random sampling number [16–18]. Thus, the results of MCM are usually treated as benchmarks. However, the adequate accuracy is possible only at the expense of a high computational cost. The FVM, proposed by Raithby and Chui, has emerged over the past two decades as a viable and robust tool for analyzing the RTE, due to its simplicity and its ability to handle complex geometries [19]. In the FVM, the energy is balanced in discrete control volumes and control angles [20]. Compared with MCM, FVM converges much faster at a similar level of accuracy.

Although the FVM has many advantages, it also has its own defects, i.e. the space and direction need to be discretized. The discrete directions are fixed and unable to be arbitrarily adjusted [21], it is thus difficult to employ FVM to calculate the radiative intensity when the unknown directions do not belong to the set of the discrete directions. For example, the radiative transfer in multilayer media with Fresnel boundary, and the radiative intensity incident into the camera in non-intrusive temperature reconstruction. Li [7,22,23] proposed a method, the so called discrete ordinates schemes with infinitely small weight (DOS+ISW), to deal with the arbitrary directional radiative intensities. In this method, the new discrete directions have no influence on the moment equations of the DOS+ISW, and can be arbitrarily arranged. At present, the most common approach for solving arbitrary directional radiative intensities is based on various angular

* Correspondence to: Key Laboratory of Energy Thermal Conversion and Control of Ministry of Education, School of Energy and Environment, Southeast University, No. 2 Sipailou, Nanjing, 210096, P. R. China.

E-mail address: chuanlongxu@seu.edu.cn (C.-L. Xu).

http://dx.doi.org/10.1016/j.jqsrt.2016.11.025
0022-4073/

Nomenclature

H	Height, m
I	the radiation intensity, $W/(m^2{\cdot}sr)$
I_b	the radiation intensity of blackbody, $W/(m^2{\cdot}sr)$
L	the length of the media, m
N	the total number
n	the refractive index
$\mathbf{n}_w$	outward normal direction of the wall
$\mathbf{r}$	the position vector
r	radius, m
S	the generalized source, $W/(m^2{\cdot}sr)$
s	the distance along a direction, m
T	the temperature, K
V_P	the volume of the control volume, m^3
x	coordinates of x component
y	coordinates of y component
z	coordinates of z component

Greeks symbols

α	the coefficient of the radial basis
β	the extinction coefficient, m-1
ε	the emissivity of the wall
Φ	the scattering phase function
ϕ	the radial basis function
φ	the azimuthal angle
κ	the absorption coefficient, m^{-1}
θ	the polar angle
σ	the Stefan-Boltzmann constant, $5.6703 \times 10^{-8} W/(m^2{\cdot}K^4)$
σ_s	the scattering coefficient, m^{-1}
ω	the scattering albedo
$\mathbf{\Omega}$	the radiation direction
Ω	the size of the solid angle, sr

Subscripts

A	position A
B	position B or bottom
i	the ith control volume
m	the mth radial basis coefficient
P	position P
r	radial
T	top
w	wall
x	x component
y	y component
z	axial
ϕ	circumferential
φ	azimuthal angle
θ	polar angle

Superscripts

*	the dimensionless term

interpolation methods [24–28]. The radial basis function is one of the most popular interpolation methods. It has many advantages, such as that the interpolation nodes can be arbitrarily scattered, it is easy to program and be extended to higher dimensional problems, etc. [29].

However, the accuracy of the interpolation is highly determined by the form of the radial basis function. Normally, the radial basis functions are just mathematical formulas and have no specific physical meaning. That is to say, the interpolation accuracy is usually not that high. In present work, we develop a new method, named Generalized Source Finite Volume Method (GSFVM), for calculating the radiative intensity of arbitrary direction. It is based on RTE and FVM. Six various test cases of 1D, 2D, and 3D radiative transfer problems are studied to verify the performance of GSFVM. The paper is organized as follows. Firstly, the GSFVM is introduced, and the computational expressions of the radiative intensity of arbitrary direction are proposed in Section 2. In Section 3, GSFVM is employed to investigate several typical test cases, and the results are presented and discussed. Finally, the main conclusions and perspectives are provided.

2. Mathematical formulation

In an enclosure filled with absorbing, emitting, and scattering gray media, the steady-state RTE at any location $\mathbf{r}$ and direction $\mathbf{\Omega}$ is given by

$$\frac{dI(\mathbf{r},\mathbf{\Omega})}{ds} = -\beta I(\mathbf{r},\mathbf{\Omega}) + n^2\kappa I_b(\mathbf{r}) + \frac{\sigma_s}{4\pi}\int_{4\pi} I(\mathbf{r},\mathbf{\Omega}')\cdot\Phi(\mathbf{\Omega},\mathbf{\Omega}')\,d\Omega' \tag{1}$$

where $I(\mathbf{r},\mathbf{\Omega})$ is the radiative intensity at location $\mathbf{r}$ and direction $\mathbf{\Omega}$; s is the distance along direction $\mathbf{\Omega}$; n denotes the refractive index of the media; $I_b(\mathbf{r})$ denotes the blackbody radiative intensity of location $\mathbf{r}$; κ, σ_s and β represent the absorption, scattering and extinction coefficients, and $\beta = \kappa + \sigma_s$; $\Phi(\mathbf{\Omega},\mathbf{\Omega}')$ is the scattering phase function. $\mathbf{\Omega}'$ is the incident direction; Ω represents the solid angle.

The generalized source item is defined as the sum of the emission enhancement and scattering enhancement items. It is described as

$$S(\mathbf{r},\mathbf{\Omega}) = (1-\omega)n^2 I_b(\mathbf{r}) + \frac{\omega}{4\pi}\int_{4\pi} I(\mathbf{r},\mathbf{\Omega}')\cdot\Phi(\mathbf{\Omega},\mathbf{\Omega}')\,d\Omega' \tag{2}$$

where ω is the scattering albedo, and $\omega = \sigma_s/\beta$.

The steady-state RTE, which is in the form of a differential-integral equation, can be converted to the form of a differential equation. It is written in the following form.

$$\frac{dI(\mathbf{r},\mathbf{\Omega})}{ds} = -\beta I(\mathbf{r},\mathbf{\Omega}) + \beta S(\mathbf{r},\mathbf{\Omega}) \tag{3}$$

In order to solve this first order linear differential equation, both sides of Eq. (3) should be multiplied by a factor $\exp(\beta s)$. Furthermore, items containing $I(\mathbf{r},\mathbf{\Omega})$ need to be moved to the left side of the equation.

$$\frac{dI(\mathbf{r},\mathbf{\Omega})}{ds}\exp(\beta s) + \beta I(\mathbf{r},\mathbf{\Omega})\exp(\beta s) = \beta S(\mathbf{r},\mathbf{\Omega})\exp(\beta s) \tag{4}$$

The two items on the left side of the equation can be combined into a differential item.

$$\frac{d}{ds}\left[I(\mathbf{r},\mathbf{\Omega})\exp(\beta s)\right] = \beta S(\mathbf{r},\mathbf{\Omega})\exp(\beta s) \tag{5}$$

After that, the equation can be easily transformed into the form of separation of variables.

$$d\left[I(\mathbf{r},\mathbf{\Omega})\exp(\beta s)\right] = S(\mathbf{r},\mathbf{\Omega})\cdot d\left[\exp(\beta s)\right] \tag{6}$$