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On the neutral points in Rayleigh transfer

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ABSTRACT

In this paper we consider the dependence of the existence and position of the neutral points on the albedo of single scattering and the optical thickness in a Rayleigh scattering plane-parallel homogeneous atmospheres. We use the Chandrasekhar method of discrete ordinates and the method of approximating the Sobolev resolvent function to solve the vector equation of transfer in *l*- and *r*-representation. On the basis of many different models of Rayleigh atmospheres we show the behaviour of the neutral points while the parallel incident flux can be both unpolarized or polarized. Our calculations show with high probability that the maximum number of neutral points in a Rayleigh atmosphere is four.

1. Introduction

The problem of the brightness and polarization of the clear sunlit sky was theoretically explained by Chandrasekhar only 83 years after Lord Rayleigh had posed it in 1871 [1]. The essence of the problem is that when a parallel beam of the Sun's radiation is incident on a Rayleigh-scattering plane-parallel homogeneous atmosphere of optical thickness τ_0 in some specified direction characterized by the cosine of the incident angle θ_0 we observe that at some angles the scattered radiation is not polarized. These points in these directions are called the neutral points.

This phenomenon was first discovered by Arago above the antisun soon after he had found in 1809 that the solar light is polarized. The next neutral point was discovered by Babinet in 1840, this time it was situated above the Sun. Two years after that, from considerations of symmetry, Brewster predicted the presence of a third neutral point below the Sun and in 1846 Babinet confirmed the existence of it.

If the zenith angle of the Sun is smaller that 70° then these points are situated at the angle of 20° above and below the Sun. And when the Sun is setting and the Brewster point disappears a new neutral point appears in the antisolar direction - the Arago neutral point. When Dave and Furukawa theoretically studied optically thicker atmospheres they found yet another neutral point in the antisolar direction [2]. This point was first observed by Bernáth et al only in 2002 [3].

During the 19th century the existence of these neutral points was looked at as a peculiar property of the clear sky optics. The location and behaviour of these points was studied by Dorno in detail during seven years in the Swiss health-resort Davos [4]. And not only in the main meridian (where azimuth $\phi = 0$) but in different azimuths. As a results he obtained the Dorno diagrams, i.e the lines dividing the sky in zones with positive and negative polarization.

Van de Hulst noted that measuring the Rubenson degree of polarization outside of the main meridian is hardly justified because the angle between the planes of polarization and meridian is not zero or 90° [5]. This means that in order to get a full picture of the behaviour of polarization degree we should measure also the *U*-component of the intensity vector.

Many attempts were undertaken to explain this rather queer behaviour but only after Chandrasekhar [6] applied his powerful method of discrete ordinates for solving the vector equation of transfer it appeared that the observed and computed neutral points were accordant.

Since the neutral points have been one of the important tools in atmospheric research for more that hundred years we undertook to investigate the existence and position of neutral points as functions of the main parameters of an atmosphere: the albedo of single scattering, the optical thickness, the optical depth at the fixed optical thickness, the angle of incidence of the illuminating beam and its polarization state and the albedo of the Lambert bottom. The question - could there be more than four neutral points - was answered in the negative. Our calculations showed that four is the maximum number of neutral points. However, this statement is not absolute and it is limited to the cases considered.

To solve the problem posed we applied the method of discrete ordinates described in our paper [7]. The code is able to tackle the problems of Rayleigh scattering in conservative and non-conservative homogeneous atmospheres at any optical depth with Lambert surface at the bottom or not.

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2. The fundamental equation

The equation of transfer for a Rayleigh scattering plane-parallel homogeneous atmosphere is the following

$$\mu\frac{\partial\mathbf{I}(\tau,\mu,\mu_0,\phi,\tau_0)}{\partial\tau}+\mathbf{I}(\tau,\mu,\mu_0,\phi,\tau_0)=\mathbf{B}(\tau,\mu,\mu_0,\phi,\tau_0), \tag{1}$$

where the vector source function is

$$\mathbf{B}(\tau,\mu,\mu_0,\phi,\tau_0)=\frac{\lambda}{4}\mathbf{Z}(\tau;\mu,\phi;\mu_0,\phi_0)\mathbf{F}e^{-\tau/\mu_0}+\frac{\lambda}{4\pi}\int_0^{2\pi}\int_{-1}^{+1}\mathbf{P}(\tau;\mu,\phi;\mu',\phi')\mathbf{I}(\tau,\mu',\mu_0,\phi',\tau_0)\mathrm{d}\mu'\,\mathrm{d}\phi'. \tag{2}$$

Throughout this paper we use the Chandrasekhar set of the Stokes parameters, i.e.

$$\mathbf{I}=(I_l,I_r,U,V)^T, \tag{3}$$

where "T" stands for "transposed".

If the atmosphere is bounded by a Lambert reflector with albedo A and there is no incident diffuse radiation at $\tau=0$, the boundary conditions for Eq. (1) are

$$\begin{aligned}&\mathbf{I}(0,\mu,\mu_0,\phi,\tau_0)=0,\\&\mathbf{I}(\tau_0,-\mu,\mu_0,\phi,\tau_0)=A\mu_0\exp(-\tau_0/\mu_0)\mathbf{EF}\\&\quad+\frac{1}{\pi}A\mathbf{E}\int_0^{2\pi}\mathrm{d}\phi'\int_0^1\mu'\,\mathrm{d}\mu'\mathbf{I}(\tau_0,\mu',\mu_0,\phi',\tau_0),\quad\mu>0,\end{aligned} \tag{4}$$

where

$$\mathbf{E}=\frac{1}{2}\begin{pmatrix}1&1&0&0\\1&1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} \tag{5}$$

and the net flux of the incident parallel beam of radiation on the atmosphere in the direction (μ_0,ϕ_0) is

$$\mathbf{F}=(F_l,F_r,F_u,F_v)^T. \tag{6}$$

In Eqs. (1)–(3) τ is the optical depth measured from the top of the atmosphere, τ_0 is the optical thickness of the atmosphere $(0\le\tau\le\tau_0)$, λ is the albedo of the single scattering $(0\le\lambda\le1)$, ϕ is the azimuth angle measured from an arbitrary point counterclockwise when looking in the positive τ direction $(0\le\phi\le2\pi)$ and μ is the cosine of the angle between the direction of travel of a photon and the positive τ-axis $(-1\le\mu\le1)$.

Chandrasekhar has shown that we may reduce the solution of the vector Eq. (1) to the solution of one two-component vector equation and three scalar equations [6]. The vector equation was solved by discrete ordinate method in [7] and the respective code may be obtained from the author. The three scalar equations of the Chandrasekhar pseudo-problem type were solved by approximating the kernel in the integral equation for the Sobolev resolvent function by a Gaussian sum. This approach allows to solve the equation for the resolvent function exactly and the result is a sum of exponentials. The parameters of this sum can be simply found, e.g. in [8]. In the present paper the order of Gauss quadrature is $N=100$.

Next we determined the Rubenson degree of polarization (RDP) as

$$P^{\pm}=\frac{I_r(\tau,\pm\mu,\mu_0,\phi,\tau_0)-I_l(\tau,\pm\mu,\mu_0,\phi,\tau_0)}{I_r(\tau,\pm\mu,\mu_0,\phi,\tau_0)+I_l(\tau,\pm\mu,\mu_0,\phi,\tau_0)} \tag{7}$$

and searched for the zeros of this function, using the simple linear interpolation. We believe that this interpolation together with rather high order of quadrature secures the accuracy at least three significant figures.

3. Results

3.1. Unpolarized incident beam

First we consider the case with unpolarized incident beam and the downward diffuse radiation only in the sun's vertical. We found the neutral curves in the (μ,τ_0) plane for different μ and λ. For a conservative atmosphere the neutral curves for small μ_0 both for solar and antisolar directions are approximately symmetric and the "bubbles" are of equal area. The larger the angle $\arccos\mu_0$ the smaller are the bubbles in the antisolar direction and they vanish completely if $\mu_0>0.448$. Fig. 1 shows the "curve of vanishing" as a function of μ_0 and λ. It appears that the smaller the albedo of single scattering the smaller is also the angle $\arccos\mu_0$ at which the bubble disappears, e.g. for $\lambda=0.5$ the neutral points in the direction of antisun disappear at $\mu_0>0.263$.

For a conservative atmosphere the neutral bubbles become larger with the growth of μ_0 in solar direction and they "take off" from the $\mu=0$ axis. They flatten in this process and if $\mu_0=1$ the neutral points disappear Fig. 2.

There are no neutral points in a conservative atmosphere for the upward radiation in the direction of antisun.

In an optically thick ($\tau_0=5$) conservative atmosphere we studied also the dependence of the neutral curves on the incident beam angle in the (μ,τ) plane. For the downward diffuse radiation we could observe only two neutral points at different angles of the incident beam - Figs. 3 and 4.

In the case of upward radiation the behaviour of the neutral curves is somewhat different.

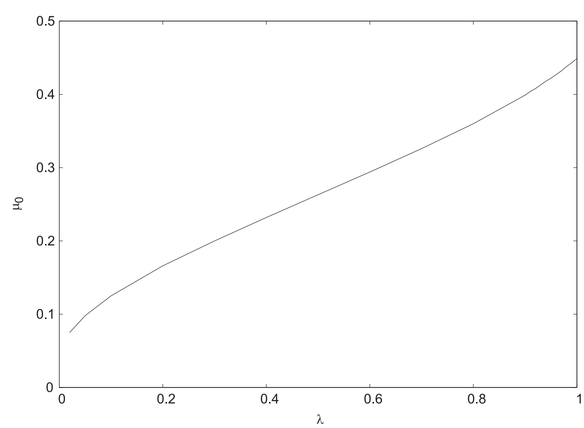

Fig. 1. There are no neutral points in (λ,μ) plane for the transmitted radiation in a Rayleigh scattering atmosphere upwards of this line ($\phi=\pi$, $F_l=F_r=0.5$, $A=0$).

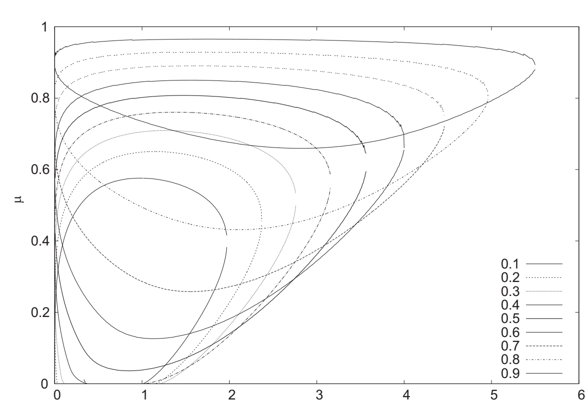

Fig. 2. The neutral curves in (τ_0,μ) plane for the transmitted radiation in a conservative Rayleigh scattering atmosphere ($\mu_0=0.1,\ldots,0.9$, $\phi=0$, $F_l=F_r=0.5$, $A=0$).