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Gaussian beam scattering by a gyrotropic anisotropic object

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A B S T R A C T

An exact semi-analytical solution is presented to the scattering of an on-axis Gaussian beam incident on a gyrotropic anisotropic object. The on-axis incident Gaussian beam, scattered fields as well as internal fields are expanded in terms of appropriate spherical vector wave functions, and the unknown expansion coefficients of the scattered fields are determined by virtue of Schelkunoff's equivalence theorem and electromagnetic boundary conditions. Numerical results of the normalized differential scattering cross section are presented, and the scattering characteristics are discussed concisely.

1. Introduction

The electromagnetic (EM) properties of anisotropic media have attracted more and more attention for their wide applications in such areas as chemistry, biology, optics, microwave technology, etc. In the investigation of the interaction between the EM waves and anisotropic media, undoubtedly it is a fundamental problem to study the EM scattering by arbitrarily shaped anisotropic objects. Some numerical methods, such as the method of moments [1], generalized multipole technique [2], integral equation technique [3], and frequency domain finite difference method [4], have been proposed to analyze the EM scattering by anisotropic bodies. In contradistinction to these numerical methods, exact analytical or semi-analytical solutions usually play an important role in providing a standard of accuracy evaluation. By using the vector wave eigenfunctions expansions, exact analytical solutions have been presented to the EM scattering by an anisotropic circular cylinder [5], uniaxial anisotropic sphere [6,7], and gyrotropic anisotropic sphere [8,9]. However, these studies are only limited to the cases of infinite circular cylinders and spheres. Recently, Schmidt et al. formulated the EM scattering by a biaxial anisotropic particle with the extended boundary condition method (EBCM) [10–12]. This paper, based on a combination of the EBCM and generalized Lorenz-Mie theory (GLMT) [13,14], is devoted to the description of an exact semi-analytical solution to the on-axis Gaussian beam scattering by an arbitrarily shaped gyrotropic anisotropic object.

The body of this paper is organized as follows. Section 2 provides the theoretical procedure for the determination of the scattered fields of an on-axis Gaussian beam by a gyrotropic anisotropic object. In Section 3, numerical results of on-axis Gaussian beam scattering properties are presented for a gyrotropic spheroid and circular cylinder of finite length. The work is summarized in Section 4.

2. Formulation

2.1. Expansions of on-axis Gaussian beam, scattered and internal fields in terms of the spherical vector wave functions

As shown in Fig. 1, an arbitrarily shaped gyrotropic anisotropic object is attached to the Cartesian coordinate system $O\ x\ y\ z$. An incident Gaussian beam propagates in free space and along the axis $O'z'$ in the plane xOz, with the middle of its beam waist located at origin O' on the axis $O'z'$. Origin O has a coordinate z_0 on the axis $O'z'$ (on-axis case), and the angle made by the axis $O'z'$ with the axis Oz

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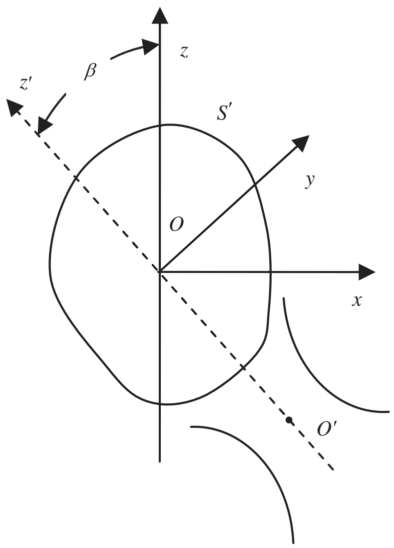

Fig. 1. A gyrotropic anisotropic object illuminated by an on-axis Gaussian beam.

is β (oblique incidence). In this paper, a time dependence of the form $\exp(-i\omega t)$ is assumed and suppressed for the EM fields.

In [15], we have obtained an expansion of the EM fields of an on-axis incident Gaussian beam (focused TEM_{00} mode laser beam) in terms of the spherical vector wave functions (SVWFs) with respect to the system $Oxyz$[16].

For the TE mode

$$\mathbf{E}^i = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} (-1)^m \frac{2n+1}{n(n+1)} \times \left[G_{n,TE}^m \mathbf{M}_{mn}^{(1)}(k_0\mathbf{r}) + G_{n,TM}^m \mathbf{N}_{mn}^{(1)}(k_0\mathbf{r}) \right] \tag{1}$$

$$\mathbf{H}^i = -iE_0 \frac{1}{\eta_0} \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} (-1)^m \frac{2n+1}{n(n+1)} \times \left[G_{n,TE}^m \mathbf{N}_{mn}^{(1)}(k_0\mathbf{r}) + G_{n,TM}^m \mathbf{M}_{mn}^{(1)}(k_0\mathbf{r}) \right] \tag{2}$$

where $\eta = \sqrt{\mu_0/\varepsilon_0}$ and k_0 are the free space wave impedance and wave number, and $G_{n,TE}^m$, $G_{n,TM}^m$ are Gaussian beam shape coefficients

$$\left(G_{n,TE}^m, \quad G_{n,TM}^m \right) = i^n \frac{(n-m)!}{(n+m)!} g_n \left(\frac{dP_n^m(\cos\beta)}{d\beta}, \quad m\frac{P_n^m(\cos\beta)}{\sin\beta} \right) \tag{3}$$

where g_n, when the Davis-Barton model of the Gaussian beam is used [17], can be described by a simpler expression known as the localized approximation [13,14]

$$g_n = \frac{1}{1+2isz_0/w_0} \exp(ik_0z_0) \exp\left[\frac{-s^2(n+1/2)^2}{1+2isz_0/w_0} \right] \tag{4}$$

where $s = 1/(k_0w_0)$, and w_0 is the beam waist radius.

For the TM mode, the corresponding expansions can be obtained only by replacing $G_{n,TE}^m$ in Eqs. (1) and (2) with $iG_{n,TM}^m$, and $G_{n,TM}^m$ with $-iG_{n,TE}^m$.

The scattered fields can be correspondingly expanded in terms of the SVWFs as follows:

$$\mathbf{E}^s = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} (-1)^m \frac{2n+1}{n(n+1)} \times [\alpha_{mn} \mathbf{M}_{mn}^{(3)}(k_0\mathbf{r}) + \beta_{mn} \mathbf{N}_{mn}^{(3)}(k_0\mathbf{r})] \tag{5}$$

$$\mathbf{H}^s = -iE_0 \frac{1}{\eta_0} \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} (-1)^m \frac{2n+1}{n(n+1)} \times [\alpha_{mn} \mathbf{N}_{mn}^{(3)}(k_0\mathbf{r}) + \beta_{mn} \mathbf{M}_{mn}^{(3)}(k_0\mathbf{r})] \tag{6}$$

where α_{mn} and β_{mn} are the unknown expansion coefficients to be determined.

The constitutive relations of the medium of a gyrotropic anisotropic object in Fig. 1 are described by a permittivity tensor $\bar{\varepsilon} = \varepsilon_1\hat{x}\hat{x} - i\varepsilon_2\hat{x}\hat{y} + i\varepsilon_2\hat{y}\hat{x} + \hat{y}\hat{y}\varepsilon_1 + \hat{z}\hat{z}\varepsilon_3$ in the system $Oxyz$ and a scalar permeability μ_0 (free-space permeability). As discussed in [18], the EM fields within the gyrotropic anisotropic object can be expanded as

$$\mathbf{E}^w = E_0 \sum_{p=-\infty}^{\infty} \sum_{q=|p|}^{\infty} [F_{pq1}\mathbf{X}_{pq1}^e(k_1\mathbf{r}) + F_{pq2}\mathbf{X}_{pq2}^e(k_2\mathbf{r})] \tag{7}$$

$$\mathbf{H}^w = -iE_0 \frac{1}{\eta_0} \sum_{p=-\infty}^{\infty} \sum_{q=|p|}^{\infty} [F_{pq1}\mathbf{X}_{pq1}^h(k_1\mathbf{r}) + F_{pq2}\mathbf{X}_{pq2}^h(k_2\mathbf{r})] \tag{8}$$

where the basis vector functions $\mathbf{X}_{pqt}^e(k_t\mathbf{r})$, $\mathbf{X}_{pqt}^h(k_t\mathbf{r})$ $(t=1,2)$ are expressed in terms of the SVWF expansion form

$$\mathbf{X}_{pqt}^e(k_t\mathbf{r}) = \sum_{l=|p|}^{\infty} \int_0^{\pi} [A_{plt}^e(\theta_k)\mathbf{M}_{pl}^{(1)}(k_t\mathbf{r}) + B_{plt}^e(\theta_k)\mathbf{N}_{pl}^{(1)}(k_t\mathbf{r}) + C_{plt}^e(\theta_k)\mathbf{L}_{pl}^{(1)}(k_t\mathbf{r})]P_q^p(\cos\theta_k)k_t^2 \sin\theta_k d\theta_k \tag{9}$$

$$\mathbf{X}_{pqt}^h(k_t\mathbf{r}) = \sum_{l=|p|}^{\infty} \int_0^{\pi} \frac{k_t}{k_0} \Big[A_{plt}^e(\theta_k)\mathbf{N}_{pl}^{(1)}(k_t\mathbf{r}) + B_{plt}^e(\theta_k)\mathbf{M}_{pl}^{(1)}(k_t\mathbf{r})]P_q^p(\cos\theta_k)k_t^2 \sin\theta_k d\theta_k \tag{10}$$

where the wave numbers k_t $(t=1,2)$ are

$$k_1^2 = \frac{B+\sqrt{B^2-4AC}}{2A}, k_2^2 = \frac{B-\sqrt{B^2-4AC}}{2A} \tag{11}$$

$$A = a_1^2\sin^2\theta_k + a_3^2\cos^2\theta_k \tag{12}$$

$$B = (a_1^4 - a_2^4)\sin^2\theta_k + a_1^2a_3^2(1+\cos^2\theta_k), C = a_3^2(a_1^4 - a_2^4) \tag{13}$$

$$a_1^2 = \omega^2\mu_0\varepsilon_1, \quad a_2^2 = \omega^2\mu_0\varepsilon_2, \quad a_3^2 = \omega^2\mu_0\varepsilon_3 \tag{14}$$

and the coefficients $A_{plt}^e(\theta_k)$, $B_{plt}^e(\theta_k)$, and $C_{plt}^e(\theta_k)$, for the sake of brevity, are provided in Appendix A .

2.2. Determination of expansion coefficients of scattered fields based on the EBCM scheme

As discussed in [19], by using Schelkunoff's equivalence theorem and EM boundary conditions the surface currents in terms of the internal fields can be set up to produce null fields inside the gyrotropic anisotropic object's surface S' and actual scattered fields outside S'. The explicit